ALGORITHM OF PRISMATIC OBJECTS OPTIMAL ARRANGEMENT ON A PALLET

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Abstract

Prismatic packaging system arose from the simple need to take better advantage of the space, whether it is a storage area or space in the transport vehicle. This is known as a modular packaging system. It was necessary first to determine the unique (optimal or ideal) the size of a pallet module, as well as the size of the prismatic package (box). Today, the problem of optimal arrangement of goods on the pallet is a very common. The new algorithm of prismatic objects optimal arrangement on a pallet is presented in the paper. This is the first level of algorithm and it deals only with same oriented prismatic objects. All variants of the object optimal arrangement are discussed in this paper.

Keywords: Packaging, Pallet, Algorithm, Optimal.

1 INTRODUCTION

Pallets are widely used throughout industry, primarily for shipping of products. A pallet is the structural foundation of a unit load which allows handling and storage efficiencies. While most pallets are wooden, pallets can also be made of plastic, metal, paper, and recycled materials. Each material has advantages and disadvantages relative to the others. Palette EUR-EPAL is shown on Fig.1. Flat-pallet timber typically has a smooth flat surface with or without the lega.

Most European countries have adopted a pallet of 800x1200 mm. There are also wooden flat pallets with the following dimensions:

* 800 mm x 1000 mm,
* 1000 mm x 1000 mm,
* 1200 mm x 1600 mm, and
* 1200 mm x 1800 mm.

All horizontal wooden pallets with the aforementioned dimensions passed the International organization ISO. But pallets with dimensions 800 x 1200 mm and 1000 x 1200 mm are the most in use.

Fig. 1 Palette EUR-EPAL

Packing items (for example, boxes) into larger objects (pallets) are very common. The problem is known as the manufacturer's pallet loading if all boxes are identical. In this paper we assume that the boxes are identical and an orientation is fixed. The paper shows all variants of the object optimal allangement. The new algorithm of prismatic objects optimal arrangement on a pallet is presented in the paper, too.

2 PALLET LOADING PROBLEM

The problem of manufacturer's pallet loading is very common [1, 2]. This is the case of a manufacturer that produces goods packaged in identical boxes of size \((l, w, h)\) which are then arranged in horizontal layers on pallets of size \((L, W, H)\) (where \(H\) is the maximum height of the loading), Fig.2.

Fig. 2Pallet and box dimensions

As a general guide, the height of the load \(H\) should not exceed the longest basedimension of the pallet. Shrink- or stretchwrapping the load usually provides greater security, minimising the possibility of movement of the goods being moved. With these techniques you can safely transport loads taller than the longest base dimension of the pallet. This will result in palletised loads that are around the internal height of closed vehicles or freight containers. We assume that the boxes are available in large quantities and are orthogonally loaded on each pallet (that is, with their sides parallel to the pallet sides). If no orientation is
fixed (that is, the boxes can be placed on any of their faces \((l, w), (l, h)\) or \((w, h)\) and because of loading stability requirements, it is usual to divide the manufacturer’s pallet loading into two subproblems:

(i) for each face, the two-dimensional problem of arranging the maximum number of rectangles (faces) into the large rectangle \((L, W)\) (pallet surface) without overlapping, and

(ii) the one-dimensional problem of stacking the most valuable layers along the width \(H\) of the pallet, where the value of each layer corresponds to its number of boxes (note that this is a knapsack problem).

When boxes are not of the same size, the problem is called the distributor’s pallet loading.

In this paper we assume that the boxes are identical and an orientation is fixed. The problem consists of packing the maximum number of rectangles \((l, w), (l, h)\) and \((w, h)\) orthogonally into a larger rectangle \((L, W)\) without overlapping (as in (i) above), yielding a layer of height \(h\). We also assume that there are no constraints related to the weight, density, fragility, etc., of the cargo loading. Besides the pallet loading, the manufacturer’s pallet loading is also of concern in the package design and the truck or rail car loading. It can be classified as 2/B/ O/C according to Dychoff’s typology of cutting and packing problems [1, 3, 4].

3 PALLET LOADING VARIANTS

There are six variants of pallet loading if all boxes are identical and same pattern of loading.

**Variant 1.** The box edge \(l\) is parallel to pallet edge \(L\), and edge \(w\) is parallel to \(W\). The maximum number of boxes on the pallet is:

\[
n_{var1} = \left\lfloor \frac{L}{l} \right\rfloor \left\lfloor \frac{W}{w} \right\rfloor \left\lfloor \frac{H}{h} \right\rfloor
\]  

(1)

**Variant 2.** The box edge \(l\) is parallel to pallet edge \(L\), and edge \(h\) is parallel to \(W\). The maximum number of boxes on the pallet is:

\[
n_{var2} = \left\lfloor \frac{L}{l} \right\rfloor \left\lfloor \frac{W}{h} \right\rfloor \left\lfloor \frac{H}{w} \right\rfloor
\]  

(2)

**Variant 3.** The box edge \(w\) is parallel to pallet edge \(L\), and edge \(l\) is parallel to \(W\). The maximum number of boxes on the pallet is:

\[
n_{var3} = \left\lfloor \frac{L}{w} \right\rfloor \left\lfloor \frac{W}{l} \right\rfloor \left\lfloor \frac{H}{h} \right\rfloor
\]  

(3)

**Variant 4.** The box edge \(h\) is parallel to pallet edge \(L\), and edge \(l\) is parallel to \(W\). The maximum number of boxes on the pallet is:

\[
n_{var4} = \left\lfloor \frac{L}{h} \right\rfloor \left\lfloor \frac{W}{l} \right\rfloor \left\lfloor \frac{H}{w} \right\rfloor
\]  

(4)

**Variant 5.** The box edge \(h\) is parallel to pallet edge \(L\), and edge \(w\) is parallel to \(W\). The maximum number of boxes on the pallet is:

\[
n_{var5} = \left\lfloor \frac{L}{h} \right\rfloor \left\lfloor \frac{W}{w} \right\rfloor \left\lfloor \frac{H}{l} \right\rfloor
\]  

(5)

**Variant 6.** The box edge \(w\) is parallel to pallet edge \(L\), and edge \(h\) is parallel to \(W\). The maximum number of boxes on the pallet is:

\[
n_{var6} = \left\lfloor \frac{L}{w} \right\rfloor \left\lfloor \frac{W}{h} \right\rfloor \left\lfloor \frac{H}{l} \right\rfloor
\]  

(6)

The best variant is the one that provides the maximum number of boxes to be placed on the pallet. This is described by Eq. (7):

\[
n_{max} = \max(n_{var1}, n_{var2}, n_{var3}, n_{var4}, n_{var5}, n_{var6})
\]  

(7)

4 PALLET LOADING EXAMPLE

To demonstrate the advantage of proposed way for selecting the best pallet loading variant, two examples for pallet loading with different box dimensions in this paper are studied.

**Example 1.** The dimensions of the boxes, which will be put on the pallet, are \(l = 400\ mm, w = 300\ mm\) and \(h = 200\ mm\). The pallet dimensions are \(L = 1200\ mm\) and \(W = 800\ mm\). Using equations (1-6), the maximum number of boxes on the pallet is: \(n_{var1} = 36, n_{var2} = 48, n_{var3} = 48, n_{var4} = 48, n_{var5} = 36\) and \(n_{var6} = 48\).

The best variant or variants are the one that provides the maximum number of boxes to be placed on the pallet, using Eq (7), and that are variants 2, 3, 4 and 6. These variants are equal because they provide the same number of boxes to be put on the pallet. In this example, the variant 3 is adopted, because the boxes lying on a pallet to its larger surface, Fig. 3.

**Example 2.** The dimensions of the boxes, which will be put on the pallet, are \(l = 500\ mm, w = 400\ mm\) and \(h = 300\ mm\). The pallet dimensions are \(L = 1200\ mm\) and \(W = 800\ mm\). Using equations (1-6), the maximum number of boxes on the pallet is: \(n_{var1} = 16, n_{var2} = 12, n_{var3} = 12, n_{var4} = 12, n_{var5} = 16\) and \(n_{var6} = 12\).

The best variant or variants are the one that provides the maximum number of boxes to be placed on the pallet, using Eq (7), and that are variants 1 and 5. These variants are equal because they provide the same number of boxes to be put on the pallet. In this example, the variant 1 is adopted, because the boxes lying on a pallet to its larger surface, Fig. 4.
5 PALLET LOADING ALGORITHM

In mathematics and computer science, an algorithm is a self-contained sequence of actions to be performed. Algorithms can perform calculation, data processing and automated reasoning tasks [5, 6].

An algorithm is an effective method that can be expressed within a finite amount of space and time and in a well-defined formal language for calculating a function. Starting from an initial state and initial input (perhaps empty), the instructions describe a computation that, when executed, proceeds through a finitenumber of well-defined successive states, eventually producing "output" and terminating at a final ending state.

In computer systems, an algorithm is basically an instance of logic written in software by software developers to be effective for the intended "target" computer(s) to produce output from given input.

In this chapter, the pallet loading algorithm is developed using equations (1-7) with only one assumption \( l > w > h \).

The first part of algorithm is shown in Fig. 5, and the second part in Fig. 6.

The first part of algorithm describes calculations of all six variants, and gives us a maximum number of boxes on the pallet for each variant.

The second part of algorithm searches for the best variant of all others and print the best variant and the maximum number of boxes on the pallet for that variant.

![Fig. 4 Pallet loading example 2](image)

![Fig. 5 The first part of pallet loading algorithm](image)

![Fig. 6 The second part of pallet loading algorithm](image)
There are two conditions which have to be satisfied for selecting the best variant of pallet loading. The first condition is to have the maximum number of the boxes to be placed on the pallet. The second condition is that boxes lying on a pallet to its larger surface. All conditions are satisfied by appropriate order of conditions (If … than) in algorithm in Fig. 6. This order of conditions in algorithm can be applied only for assumption \( l > w > h \). As it was explained in previously chapter, there are cases that more variants satisfy the first condition. The best variant must match all conditions.

6 CONCLUSION

Prismatic packaging system arose from the simple need to take better advantage of the space, whether it is a storage area or space in the transport vehicle. This is known as a modular packaging system. Also, the problem is known as the manufacturer's pallet loading if all boxes are identical. As a solution, it was necessary first to determine the unique (optimal or ideal) the size of a pallet module, as well as the size of the prismatic package (box). Nowadays, the problem of optimal arrangement of goods on the pallet is a very common.

It was shown that two conditions for optimal arrangement of boxes on the pallet are exists. The first condition is to have the maximum number of the boxes to be placed on the pallet. The second condition is that boxes lying on a pallet to its larger surface. The best variant must match all conditions. As it was mentioned, the new algorithm of prismatic objects optimal arrangement on a pallet is presented in the paper. This is the first level of algorithm and it deals only with same oriented prismatic objects (boxes) and with only one assumption \( l > w > h \). All variants of the object optimal arrangement are discussed in this paper.

Two examples of boxes optimal arrangement on a pallet show all advantages of proposed algorithm. Presented algorithm is good base for further developing some program on PC for optimal arrangement boxes on a pallet as well as for solving the problem of optimal arrangement of some other objects on a pallet, for instance cylindrical objects. Also, presented algorithm is a good base for further researching optimal arrangement boxes on a pallet when boxes are not of the same size; the problem is called the distributor's pallet loading.

REFERENCES


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