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A MULTI - OBJECTIVE APPROACH TO THE LOGISTICS CENTERS LOCATIONS PROBLEM

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Abstract

Logistic service assumes delivery of logistics units in the fastest possible way along with minimization of transportation and handling costs. In order to properly organize delivery in a specific region, the logistic company should try to find the answers to the following questions: What should be the total number of delivery hubs? b) Where these facilities should be located? c) How should demand for facilities' service be allocated to facilities? By identifying the right locations of the hubs, correctly defined transportation links in the network and properly setting of transportation schedule, it is possible to reach the maximization of one defined criterion or a combination of them. These issues are modeled in this paper as a multi-objective problem. The model developed is based on Compromise Programming and Genetic Algorithms.

Key words: *Logistic Services, Hub Locations, Compromise Programming, Genetic Algorithms.*

1 INTRODUCTION

Logistic service assumes delivery of logistics units (LU) in the fastest possible way along with minimization of transportation and handling costs. Every logistic unit to be delivered is characterized by origin, destination, and usualy a delivery deadline. There are various options of the delivery service, such as courier service, guaranteed next business day delivery, guaranteed two business days delivery, etc. Logistic companies organize picking-up pallets and containers, organize transportation, as well as unloading of freight at the destination points. As a rule, delivery network in the observed country or region is not fully connected. Delivery companies usually organize hub delivery network, since flows between hubs are characterized by economies of scale effect. At hubs, LU are exchanged across vans, trucks, and planes. In other words, majority of LU are transported from one node to another without a direct service (Figure 1).

In order to properly organize delivery in a specific region, the logistic company should try to find the answers to the following questions: What should be the total number of delivery hubs? b) Where these facilities should be located? c) How should demand for facilities' service be allocated to facilities? These issues are modeled in this paper as a multi-objective problem, in order to simultaneously find the optimal number of hubs, find the optimal location for the calculated number of hubs, and allocate the population to the suggested hubs.

The total number of hubs, hub locations, and nodes allocation to hubs influence the total transportation costs, and the level of service in delivery network. The flows between all pairs of nodes (Origin-Destination matrix) represent the basic input data for the hub location problem. O'Kelly [7,8] was the first to study the hub location problem. Hub location problems are known to be very difficult ones and are generally considered to be NP-hard. Various aspects of this problem were studied, among others[1,3,9,10,11,12,13,14,15,16,17,18,19].

In this paper, we consider the case when there is no capacity constraints at hubs. We also assume that the total number of hubs in the network is not prescribed in advance. The problem studied in this paper could be defined in the following way: for known flows, and the distances between all pairs of nodes, determine the total number of hubs, hub locations, and allocate non-hub nodes to hubs in such a way to minimize total service costs, and to maximize level of service offered to the clients.

The aim of this paper is to present a multi-objective approach for solving delivery hub location problem. The considered delivery hub location problem is solved by a combination of Genetic Algorithm and Compromise Programming and is also supported by numerical examples.

The paper is organized as follows: The statement of the problem is given in Section 2. Section 3 describes proposed solution to the problem. The experimental results are given in Section 4. Conclusions and directions for future research are given in Section 5.

2 STATEMENT OF THE PROBLEM

We studied the deleivery problem for the case of a region that could be served exclusively by the fleet of vans and trucks. Thus, planes have not been included in the delivery operations considered here. Without loss of generality, we considered the serevice option known as 'guaranteed two business days delivery'. In this paper, we studied hub location problem in a case of non-oriented network, represented by a graph G = (N, A). This graph includes a set of consecutively numbered nodes N, as well as a set of consecutively numbered links A. We denote by n cardinality of set N.

Nodes denoted by H in Figure 1 are hub nodes, while nodes represented by triangles are spokes. In the majority of cases, network of hub nodes is a completely connected graph. We assume that every node is connected to one of the hubs in the network. We denote respectively by R ($R \subseteq N$) and S ($S \subseteq N$) the set of LU origin and the set of LU destination nodes. We consider the case when the total number of future hubs in the delivery network is not given in advance. We assume that each hub has a very big capacity, thus we consider uncapacitated hub location problem.

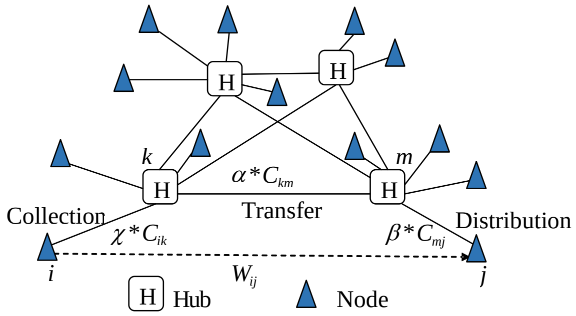

Fig. 1 *Hub-and-spoke parcel delivery network*

In this paper we assumed that optimal decisions related to the hub locations should be made in the presence of trade-offs between two or more conflicting objectives. At the same time we tried to optimize two conflicting objectives subject to specified constraints. The objectives that we considered are minimization of total costs and the maximization of level of service offered to the clients. These objectives are conflicting with each other. We propose the multi-objective formulation of the parcel delivery hub locations problem.

2.1 Measuring the total service costs

When analyzing the delivery system costs, we take into account collection costs from origins to hubs, transportation costs from hubs to hubs, distribution costs from hubs to destinations, and the costs of establishing hubs. We use notation proposed by O'Kelly [7,8] and modified by Topcuoglu et al. [17]. We denote by W_{ij} the number of units of flow between node i and node j, and by C_{ij} the transportation cost of a unit of flow between node i and node j. We assume that $C_{ii} = 0$ and $W_{ii} = 0$. Let us consider the path $i \rightarrow k \rightarrow m \rightarrow j$. Without loss of generality, we assume that all costs are symetric ($C_{ij} = C_{ji}$). The total costs of transporting the LU from origin i, through hubs k and m, to destination j equals:

$$C_{ik} + \alpha C_{km} + C_{mj} \tag{1}$$

where $\alpha \leq 1$ is a parameter that reflects the economies of scale effects in hubs' flows. Let us introduce into analysis the following binary variables:

$$x_{ik} = \begin{cases} 1 & \text{if node } i \text{ is connected with hub at } k \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and

$$x_{ii} = \begin{cases} 1 & \text{if node } i \text{ is a hub} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The total collection costs in the whole network equals:

$$\sum_i \sum_j W_{ij} \sum_k x_{ik} C_{ik} \tag{4}$$

The total transportation costs between hubs equals:

$$\sum_i \sum_j W_{ij} a \sum_k \sum_m x_{ik} x_{jm} C_{km} \tag{5}$$

The total distrubution costs in the whole network equals

$$\sum_i \sum_j W_{ij} \sum_m x_{jm} C_{jm} \tag{6}$$

We denote by f_i the cost of establishing the hub in node i. The total costs $f_1(\vec{x})$ of the parcel delivery system equals:

$$f_1(\vec{x}) = \sum_i \sum_j W_{ij} \sum_k x_{ik} C_{ik} + \sum_i \sum_j W_{ij} a \sum_k \sum_m x_{ik} x_{jm} C_{km} + \sum_i \sum_j W_{ij} \sum_m x_{jm} C_{jm} + \sum_i x_{ii} f_i \tag{7}$$

2.2 Measuring the level of service offered to clients

Guaranteed next business day delivery assumes existing of time window for picking-up parcels. For example, the operator could offer guarantees to clients that all LU collected between 8:00 a.m. and 3:00 p.m. will be delivered by the end of the next business day. Figure 2 shows time window for picking-up LU. The earliest e_i , and the latest l_i possible time point for accepting LU are denoted in the Figure 2. Time point C represents collection of i-th LU.

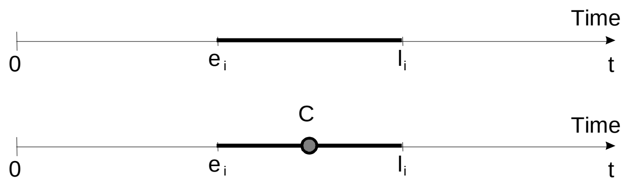

Fig. 2 *Time window for picking-up parcels*

Let us consider the path $i \rightarrow k \rightarrow m \rightarrow j$. Without lost of generality, we assume that all travel times are symetric ($t_{ij} = t_{ji}$). The total transportation time of the LU from origin i, through hubs k and m, to destination j equals:

$$t_{ij} = \sum_k x_{ik} t_{ik} + \sum_k \sum_m x_{ik} x_{jm} t_{km} + \sum_m x_{jm} t_{jm} \tag{8}$$

Let us denote by T^* guaranteed time period for making the delivery (for example, in the case of guaranteed next day delivery, $T^* = 24$ *hr*.). In other words if we want to make delivery during next business day LU must be in destination logistic center befor the wehicles left center. The latest possible time point l_i for picking-up parcels at node i must satisfy the following inequality:

$$l_i + \max_j \{t_{ij}\} \leq T^* \tag{9}$$

i.e.:

$$l_i \leq T^* - \max_j \{t_{ij}\} \tag{10}$$

$$l_i \leq T^* - \max_j \left\{ \sum_k x_{ik} t_{ik} + \sum_k \sum_m x_{ik} x_{jm} t_{km} + \sum_m x_{jm} t_{jm} \right\} \tag{11}$$

We use the time window concept to measure the level-of-service offered to clients. The wider the time windows, the higher the level-of-service. We denote by tw_i the time window for picking-up parcels at node i ($tw_i = l_i - e_i$) Without loss of generality, we assume that $e_i = 0$. The $tw_i \sum_j W_{ij}$ represents the total number of hours available for picking-up at node i. The total number of hours $f_2(\vec{l})$ available for collection in the whole network equals:

$$f_2(\vec{l}) = \sum_i tw_i \sum_j W_{ij} = \sum_i l_i \sum_j W_{ij} \tag{12}$$

The total number of nodes and/or total number of inhabitants in the network that can enjoy offered delivery services also represent important attributes of the level-of-service. The total number of hubs, hub locations, and nodes allocation to hubs influence the values of these quantities. For example, the total number of nodes (inhabitants) that can enjoy guaranteed next business day delivery could be significantly increased by introducing new hubs and/or by relocating existing hubs.

There are two possibilities for each node in the network. Node could be outside of the delivery system, or node could be connected with all other nodes in the network. We have not considered the case when node could have delivery service with limited number of other nodes in the network. Imagine that operator guarantees next business day delivery, but accepts logistic units only between 8:00 a.m. and 8:05 a.m. For the majority of potential clients such service would be unacceptable. Obviously, there is a need to establish minimum value of time interval tw_i, i = 1, 2,…, n. For example, in the case when the earliest time point is e_i = 8 a.m., minimum acceptable latest time point for accepting parcels could be l_i = 10 a.m. We denote minimum acceptable latest time point for accepting parcels by l^*. Node i could be included in the delivery system if the following is satisfied:

$$l^* \le T^* - \max_j \{t_{ij}\} \tag{13}$$

Let us introduce the following binary variables:

$$y_i = \begin{cases} 1 & when \quad l^* \le T^* - \max_j \{t_{ij}\} \\ 0 & otherwise \end{cases} \tag{14}$$

The total number of nodes $f_3(\vec{y})$ included in the parcel delivery system equals:

$$f_3(\vec{y}) = \sum_{i=1}^{n} y_i \tag{15}$$

2.3 Mathematical formulation of the problem

Here, we propose making optimal, or near optimal, decisions related to the delivery hub location problem, in the presence of trade-offs between two or more conflicting objectives. We attempted to simultaneously optimize three conflicting objectives subject to specified constraints.

The objectives that we consider are minimization of total costs, maximization of the total number of available hours for accepting logistic units, and the maximization of the total number of nodes included in the delivery system. These objectives are conflicting with each other. We propose the following mathematical formulation of the multi-objective parcels delivery hub location problem:

Minimize

$$f_1(\vec{x}) = \sum_i \sum_j W_{ij} \sum_k x_{ik} C_{ik} + \sum_i \sum_j W_{ij}\, a \sum_k \sum_m x_{ik} x_{jm} C_{km} + \sum_i \sum_j W_{ij} \sum_m x_{jm} C_{jm} + \sum_i x_{ii} f_i \tag{16}$$

Maximize

$$f_2(\vec{l}) = \sum_i l_i \sum_j W_{ij} \tag{17}$$

Maximize

$$f_3(\vec{y}) = \sum_{i=1}^{n} y_i \tag{18}$$

subject to:

$$l_i \le T^* - \max_j \left\{ \sum_k x_{ik} t_{ik} + \sum_k \sum_m x_{ik} x_{jm} t_{km} + \sum_m x_{jm} t_{jm} \right\} \tag{19}$$

$$\sum_k X_{ik} = 1 \quad \forall i \in N \tag{20}$$

$$X_{kk} - X_{ik} \ge 0 \quad \forall i,k \in N \tag{21}$$

$$X_{ik} \in 0{,}1 \quad \forall i,k \in N \tag{22}$$

$$y_i = \begin{cases} 1 when \quad l^* \le T^* - max_j \left\{ \sum_k x_{ik} t_{ik} + \sum_k \sum_m x_{ik} x_{jm} t_{km} + \sum_m x_{jm} t_{jm} \right\} \\ 0 otherwise \end{cases} \tag{23}$$

The objective function $f_1(\vec{x})$ represents the total costs of the delivery system, while the second objective function $f_2(\vec{l})$ represents the total number of hours available for collection in the whole network. The third objective function $f_3(\vec{y})$ represents the total number of nodes included in the delivery system. Constraint (19) defines the latest possible time points for collecting LU at nodes. Constraint (20) prescribes that each node is assigned to one and only one hub. Constraint (21) requires that node i is assigned to node k only if k is a hub. The other constraints describe variables of the model.

Determining hub locations as expressed by relations (16)-(23) is a multicriteria nonlinear integer programming problem of the general form:

$$\max \left\{ f_1(x),\, f_2(x), \ldots, f_r(x) \,\middle|\, x \in X \quad and \; x \; integer \right\} \tag{24}$$

where $X \subset R^n$ is the set of feasible points defined by given constraints and $f_1(x), f_2(x), \ldots, f_r(x)$ are criteria that must be maximized. If one of the criteria is to be minimized, for example $f_i(x)$, then the usual transformation is made:

$$\min f_i(x) = \max[- f_i(x)] \qquad (24)$$

Due to the conflicting nature of the given criteria, there is usually no solution that simultaneously maximizes all of the criteria. For this reason the solution to problem (16) – (23) usually comprises a Pareto-optimal (efficient, non-inferior) solution. This is the solution where no criterion can be improved without simultaneously worsening at least one of the remaining criteria. More precisely, it says that x^* is the Pareto optimal solution if there is no other $x \in X$ such that:

$$f_i(x) \geq f_i(x^*) \qquad i = 1,2,\ldots,r \qquad (25)$$

The set of Pareto-optimal solutions is a subset to feasible set X.

3 PROPOSED SOLUTION TO THE PROBLEM

For integer problems that have a finite, but enormous number of feasible points, an approximate solution to the given problem is found by examining a randomly chosen subset of feasible points. The variables of the problem randomly take on discrete values from the intervals in which they are defined and thereby form a trial point whose feasibility should be examined. The point that gives the best criterion value is determined among the generated feasible points and it is considered to be an approximate optimal solution to the problem.

We use compromise programming [2] as a tool for solving multi-objective parcel delivery hub location problem.

The vector $[f_1^o, f_2^o, \ldots, f_k^o]$ is called the ideal vector, where f_i^o(i=1,2,...K) denotes the optimum of the *i*-th objective function. The point that determines the ideal vector is called the ideal point. In real-life applications, it is rare, if not impossible, to discover the ideal solution of the considered multi-objective problem. Duckstein [4] proposed the following measure of "possible closeness to ideal solution".

$$L_p = [\sum_{i=1}^{K} w_i^p \left| \frac{f_i(\vec{x}) - f_i^o}{f_{i\,worst} - f_i^o} \right|^p]^{\frac{1}{p}} \qquad (26)$$

where

$f_i(\vec{x})$ – *i*-th objective function value that is result of implementing decision $\vec{x}$

f_i^o – the optimum value of the *i*-th objective function

$f_{i\ worst}$ – the worst value obtainable for the *i*-th objective function

K – total number of objective functions

w_i – *i*-th objective function's weight

p – the value that shows distance type: for $p = 1$, all deviations from optimal solutions are in direct proporsion to their size, while $2 \leq p \leq \infty$, bigger deviation carry larger weight in L_p metric.

We can generate various compromise solutions, by choosing different parameter values. In this way, we are able to present several feasible alternatives to decision-maker.

The optimal values f_i^o defined objective functions should be discovered in order to calculate the closeness to ideal solution L_p. In other words, it is necessary to solve single-objective parcel delivery hub location problem for every defined objective function. When solving single-objective problems we use an approach based on Genetic Algorithms.

3.1 Genetic Algorithm Approach to the Single-Objective delivery Hub Location Problem

In the first step, we obtain optimal values of the specific objective functions f_i^o by genetic algorithm. Genetic algorithms [5,6] represent search techniques used for solving complex combinatorial optimization problems. These algorithms were developed by analogy with Darwin's theory of evolution and the basic principle of the "survival of the fittest." In the case of genetic algorithms, as opposed to traditional search techniques, the search is run in parallel from a population of solutions. At first, various solutions to the considered maximization (or minimization) problem are generated. In the next step, the evaluation of these solutions, that is, the estimation of the objective (cost) function is made. Some of the "good" solutions yielding a better „fitness" are further considered. The remaining solutions are eliminated from consideration. The chosen solutions undergo the phases of reproduction, crossover and mutation. After that, a new generation of solutions is produced, followed by a new one, and so on. Each new generation is expected to be "better" than the previous one. The production of new generations ceases when a prespecified stopping condition is satisfied. The final solution of the considered problem is the best solution generated during the search.

3.2 String representation and Initial Population Generation

Our chromosomes contain full information about solutions they represent. We use binary strings to represent solutions. Each bit in the string offers information about hub location in a specific node. Figure 3 shows two examples of the binary strings. First string (Parent 1) represents solutions with eight hubs. Hubs are located in nodes: 1, 4, 5, 8, 11, 12, 14 and 15.

We generate initial population of solutions in a random manner. In the first step, we randomly generate number of hubs (number of 1's) for every binary string. The number of hubs H in any string must be within the interval $0 < H \leq n$. In the second step, we randomly generate hub locations (locations of 1's within the string). The probability that the node *i* will be selected to be a hub equals:

$$p_i = \frac{U_i}{\sum_j U_j} \qquad (27)$$

where:

$U_i = O_i + D_i$

O_i – the total number of LU originating from node *i*

D_i – the total number of LU whose destination is node *i*

The quantity U_i represents the total number of "operations" (receiving and shipping logistic units) in node *i*. The higher the total number of operations in the node, the higher the probability for a node to be selected as a hub.

Once the hub locations were known, the allocation of non-hub nodes to hubs was performed. The allocation of non-hub nodes to hubs could be performed in a variety of ways. We assigned every non-hub node to its nearest hub. Once the allocation was finished, it was possible to calculate fitness function value of every generated solution.

3.3 Selection, Crossover and Mutation

In the first step, we copied the best chromosomes to a new population (we usually copied 10% ÷ 20% of the best chromosomes). The remaining chromosomes from the parent population were selected in a random manner. The probability that chromosome *i* will be selected to be the parent (in the case of maximization problem) equals:

$$\mathrm{p}_i = \frac{f_i}{\sum_j f_j} \tag{28}$$

where:

f_i – fitness function value of the *i*-th chromosome

In other words, we use well-known roulette wheel selection. The higher the fitness function value, the higher the probability for a node to be selected as a parent.

The crossover probability in this paper equals 90%. We used uniform crossover (Figure 3). Bits were copied in a random manner from the first or from the second parent.

Parent 1	**1 0 0** 1 **1 0** 0 1 0 **0** 1 **1** 0 1 **1**
Parent 2	1 1 0 **0** 1 1 **1 0 0** 0 **0** 1 **1 1** 1
Offspring 1	**1 0 0 0 1 0 1 0 0 0 0 1 1 1 1**
Offspring 2	1 1 0 1 1 1 0 1 0 0 1 1 0 1 1

Fig. 3 *Example of uniform crossover in the case of network with 15 nodes*

Only offsprings that contain at least one hub were considered for further analysis.

Parts of chromosome could be mutated. In our case, mutation refers to the change in value from 1 to 0 or vice versa. The probability of mutation was very small (0.75%). The purpose of mutations is to prevent an irretrievable loss of genetic material at some points along the string. For example, in the overall population a particularly significant bit of information might be missing (for example, none of the strings have 0 at the sixth position), which can considerably influence the determination of the optimal or near-optimal solution. Without mutation, none of the strings in future populations could have 0 at the sixth position.

Once the crossover and mutation were performed, it was possible to calculate fitness function value of every generated solution.

4 EXPERIMENTAL RESULTS

Numerous numerical experiments were performed. The code is written in Matlab. All of our experimental studies have been conducted on Pentium IV 2.2 GHz, with a RAM memory size of 1 GB (under Windows XP).

4.1 Experimental results: single-objective case

We first show experimental results related to the minimization of the total costs of the delivery system. The number of generations varied between 20 and 1,000. In most of the considered cases, the obtained results converged after 100 generations. The coeficient α varied within the interval [0.2, 1]. The cost of establishing the hub *f* varied within the interval [0, 80]. The input data were taken from paper of Topcuoglu et al. (2005). The considered network contains 81 nodes. There were 80 individuals in every generation. The total number of generations was equal to 100.

The obtained results related to the single-objective problem of costs minimization are given in Table 1.

Table 1 Results obtained in the case of single-objective problem of costs minimization

α	*f*	Total costs	Hub locations	CPU [sec]
0,2	0	88.42	1,6,7,21,23,25,27,34,35,38, 42,44,52,55,58,60,61,63,65,66	56.83
	20	235.06	1,6,21,25,34	28.81
	40	313.78	34	27.67
	60	333.78	34	26.61
	80	353.78	34	26.48
0,4	0	119.56	1,6,21,25,27,34,38,44,52,55,58, 60,61,63,65	49.73
	20	253.46	1,6,21,25,34	29.70
	40	313.78	34	26.45
	60	333.78	34	25.61
	80	353.78	34	25.70
0,6	0	143.96	1,6,21,25,27,34,38,44,52,55, 58,60,61,63	43.81
	20	271.86	1,6,21,25,34	30.56
	40	333.78	34	27.56
	60	353.78	34	28.17
	80	373.78	34	28.98
0,8	0	166.46	1,6,21,25,27,34,38,44,52,55,58, 60,61,63	38.19
	20	290.26	1,6,21,25,34	30.97
	40	313.78	34	28.81
	60	333.78	34	29.11
	80	353.78	34	29.14
1	0	188.96	1,6,21,25,27,34,38,44,52, 55,58,60,61,63	49.83
	20	293.78	34	21.72
	40	313.78	34	19.83
	60	333.78	34	19.73
	80	353.78	34	19.39

The total number of hubs, as well as hub locations, highly depends on parameter *α* and parameter *f* values. The lower the *α* values, the more hubs exist in the network. The higher the *f* values, the less hubs exist in the network. Figure 4 shows fitness function values through generatrions for the case when α = 0.6 and *f* = 20.

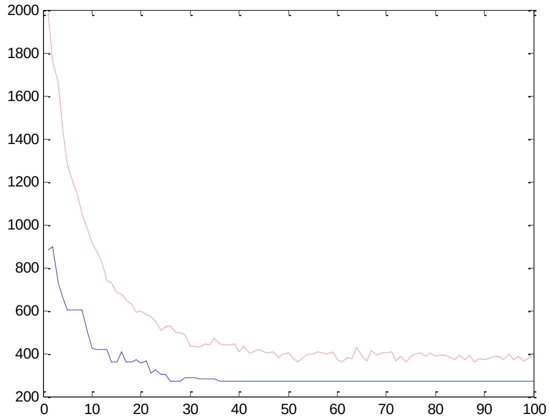

Fig. 4 *Fitness function values through generations (α = 0.6; f = 20)*

The full line represents the best value in every generation, while the dashed line depicts the average fitness function values. The changes of the fitness function values through generations were very similar in all other experiments.

4.2 Experimental results: multi-objectives case

Solving the multicriteria problem (16) – (23) by the Compromise Programming first requires an ideal point. Therefore, we solved three single criterion problems of determining hub locations with the following objective functions: minimization of the total costs, maximization of the total number of available hours for accepting LU, and the maximization of the total number of nodes included in the delivery system. We minimized possible closeness to the ideal solution $L_p = [\sum_{i=1}^{K} w_i^p \left| \frac{f_i(\vec{x}) - f_i^o}{f_{i\,\max} - f_i^o} \right|^p]^{\frac{1}{p}}$ by proposed Genetic Algorithm. We solved the problem several times for various pairs of criteria weights. We denote respectively by w_1, w_2, w_3 the criteria weights. We varied the weights within the interval [0,1] by step 0.1.

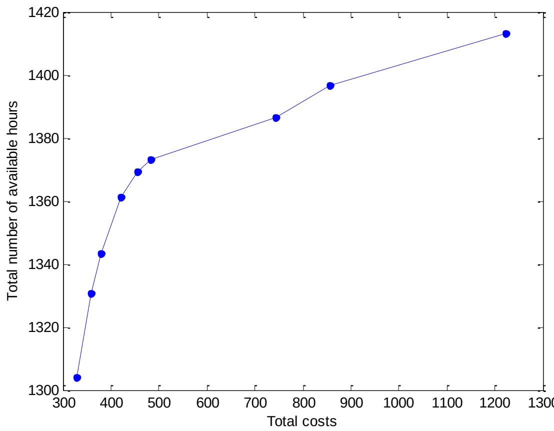

Fig. 5 *Pareto frontier in the two-objective case (minimization of the total costs, and maximization of the total number of available hours for accepting parcels and packages)*

Figure 5 shows Pareto frontier (α=0.9; f=20) for the two-objective case (minimization of the total costs, and maximization of the total number of available hours for accepting LU).

Table 2 Results obtained in the case of two-objective functions

Total Costs $f_1(\vec{x})$	The total number of available hours for accepting logistic units $f_2(\vec{l})$	w_1	w_2	Hub locations
327.7	1304	0.9	0.1	1,6,21,25,55
357.7	1330.7	0.8	0.2	1,6,21,25,38,55,61
378.9	1343.3	0.7	0.3	1,6,21,25,38,44,52,55
420.7	1361.3	0.6	0.4	1,6,21,25,38,42,44,55,61
454.8	1369.3	0.5	0.5	1,6,21,25,27,38,42,44,55,58,61
484.1	1373.3	0.4	0.6	1,6,19,21,25,27,38,42,44,55,58,61
744.2	1386.5	0.3	0.7	1,6,7,10,16,20,21,25,27,34,35,38, 42,44,45,52,55,59,60
856.5	1396.5	0.2	0.8	1,3,6,7,10,20,21,24,25,27,34,35, 38,41,42,44,45,52,55,59,60
1222	1413.3	0.1	0.9	1,3,6,7,10,20,21,24,25,26,27,31, 34,35,38,41,42,44,45,46,51,52, 55,59,60,66,67,72,73

The points represent feasible solutions that we generated. None of the shown points on the frontier are strictly dominated by any other. These solutions are Pareto optimal, because there are no other solutions that are superior in all objective. Varying the weight values enables the generation of a large number of solutions that facilitate the decision maker's understanding of the problem and the choice of a final solution.

We also tested the proposed approach in the case of three objective functions, when delivery is guaranteed within 24 hours. The obtained results are shown in Fig. 6 and Table 3.

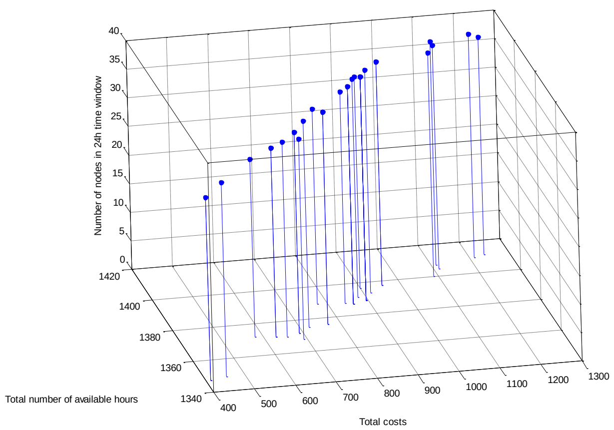

Fig. 6 *Non-dominated solutions in the three-objective case (minimization of the total costs, maximization of the total number of available hours for accepting LU and maximization of the total number of nodes included in the delivery system)*

Table 3 - Results obtained in the case of three-objective functions[1]

Total Costs $f_1(\vec{x})$	The total number of available hours for accepting parcels $f_2(\vec{l})$	The total number of nodes included in the delivery system $f_3(\vec{y})$	w_1	w_2	w_3	Hub loctions
953.73	1397.3	39	0.1	0.1	0.8	1,3,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,60,61
895.53	1388.8	39	0.1	0.2	0.7	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,61
1088.2	1400.2	39	0.1	0.3	0.6	1,3,6,7,9,10,14,20,21,24,25,27,29,34,35,37,38,41,42,44,45,48,51,52,55,59,61,67
953.73	1397.3	39	0.1	0.4	0.5	1,3,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,60,61
1111.4	1404.6	39	0.1	0.5	0.4	1,3,6,7,9,10,14,20,21,24,25,27,29,34,35,37,38,41,42,44,45,48,51,52,55,59,60,61,67
1111.8	1407	39	0.1	0.6	0.3	1,3,6,7,10,14,20,21,24,25,27,29,31,34,35,37,38,41,42,44,45,46,51,52,55,59,60,61,67
1211.4	1409.9	39	0.1	0.7	0.2	1,3,6,7,10,16,20,21,24,25,26,27,29,31,34,35,37,38,42,44,45,46,51,52,55,59,60,61,72,73
1239	1411.2	38	0.1	0.8	0.1	1,2,3,6,7,10,16,19,20,21,24,25,26,29,31,34,35,38,42,44,45,46,51,52,55,59,60,61,63,66,72,73
895.53	1388.8	39	0.2	0.1	0.7	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,61
895.53	1388.8	39	0.2	0.2	0.6	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,61
895.53	1388.8	39	0.2	0.3	0.5	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,61
895.53	1388.8	39	0.2	0.4	0.4	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,61
917.43	1393.1	39	0.2	0.5	0.3	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,60,61
881.3	1391.4	38	0.2	0.6	0.2	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,52,55,59,60,61
899.28	1396.5	37	0.2	0.7	0.1	1,3,6,7,10,16,20,21,24,25,27,34,35,37,38,42,44,45,51,52,55,59,60
895.53	1388.8	39	0.3	0.1	0.6	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,61
895.53	1388.8	39	0.3	0.2	0.5	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,51,52,55,59,61
859.45	1387.1	38	0.3	0.3	0.4	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,52,55,59,61
859.45	1387.1	38	0.3	0.4	0.3	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,52,55,59,61
842.27	1388	37	0.3	0.5	0.2	1,6,7,10,16,20,21,24,25,27,34,35,37,38,42,44,45,51,52,55,59
776.78	1389.1	34	0.3	0.6	0.1	1,6,7,10,16,20,21,24,25,27,34,35,38,42,44,45,52,55,59,60
859.45	1387.1	38	0.4	0.1	0.5	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,52,55,59,61
859.45	1387.1	38	0.4	0.2	0.4	1,6,7,10,16,20,21,24,25,27,29,34,35,37,38,42,44,45,52,55,59,61
769.93	1376.1	37	0.4	0.3	0.3	1,6,7,16,20,21,24,25,27,29,34,35,37,38,42,44,51,52,55,61,62
689.89	1372.1	35	0.4	0.4	0.2	1,6,7,16,20,21,25,27,28,34,35,37,38,42,44,51,55
629.52	1370.8	33	0.4	0.5	0.1	1,6,7,16,20,21,25,27,34,35,37,38,42,44,52,55
769.93	1376.1	37	0.5	0.1	0.4	1,6,7,16,20,21,24,25,27,29,34,35,37,38,42,44,51,52,55,61,62
689.89	1372.1	35	0.5	0.2	0.3	1,6,7,16,20,21,25,27,28,34,35,37,38,42,44,51,55
689.89	1372.1	35	0.5	0.3	0.2	1,6,7,16,20,21,25,27,28,34,35,37,38,42,44,51,55
580.93	1369.2	31	0.5	0.4	0.1	1,6,7,16,20,21,25,27,34,35,38,42,44,52,55
720.31	1375.1	36	0.6	0.1	0.3	1,6,7,16,20,21,24,25,27,34,35,37,38,42,44,51,52,55,62
656.69	1370.4	34	0.6	0.2	0.2	1,6,7,16,20,21,25,27,28,34,35,37,38,42,44,55
453.03	1348.7	34	0.6	0.3	0.1	1,6,21,25,27,28,37,38,44,55
629.52	1370.8	33	0.7	0.1	0.2	1,6,7,16,20,21,25,27,34,35,37,38,42,44,52,55
409.94	1347.2	32	0.7	0.2	0.1	1,6,21,25,27,28,38,44,55
409.94	1347.2	32	0.8	0.1	0.1	1,6,21,25,27,28,38,44,55

[1] α=0.9; f=20; K=3; p=2;

$$L_p = \left[\sum_{i=1}^{K} w_i^p \left| \frac{f_i(\vec{x}) - f_i^o}{f_{i\,\max} - f_i^o} \right|^p \right]^{\frac{1}{p}} = \left[\sum_{i=1}^{3} w_i^2 \left| \frac{f_i(\vec{x}) - f_i^o}{f_{i\,\max} - f_i^o} \right|^2 \right]^{\frac{1}{2}}$$

$f_1^o = 294$; $f_{1max} = 5675$ $f_2^o = 1432$; $f_{2max} = 0$; $f_3^o = 39$; $f_{3max} = 0$

Number of individuals in every generation = 500, Number of generations = 200

5 CONCLUSION

The delivery hub location problem was studied in the paper. When determining hub locations, the interests of both the operator and the client must be taken into consideration. Due to the conflicts of these interests, the task of determining hub locations is formulated as a multi-criteria decision making problem, as opposed to the typical approach under which one chosen criterion is optimized, while disregarding alternative ones. We have made an attempt to make good decisions related to the delivery hub locations problem, in the presence of trade-offs between two or more conflicting objectives. We determined the total number of hubs, and hub locations, and we allocated non-hub nodes to hubs in such a way to simultaneously minimize total service costs, and to maximize level of service offered to clients. The objectives considered in the paper were minimization of the total delivery costs, maximization of the total number of available hours for accepting logistic units, and the maximization of the total number of nodes included in the delivery system. The mathematical formulation of the problem considered is proposed in the paper.

We used combination of Compromise Programming and Genetic Algorithms in order to simultaneously optimize three conflicting objectives, subject to specified constraints. We performed a computational study to test the performance of the proposed approach. The criteria weight values were varied, which allowed us to generate a large number of solutions for decision-makers.

The proposed model has been implemented on a large-scale network. The network consisted of 81 nodes.

The basic input data for the problem considered are the estimated number of logistic units between pairs of cities. It is often impossible to estimate these numbers with enough precision (i.e. there is a degree of uncertainty surrounding the numbers of units between pairs of cities). Uncertainty is also frequent in the estimation of future operator's costs. A future research direction would be developing the model that incorporates uncertainty in numbers of logistic units, as well as operator's costs. The basic goal of future research should be to present the fuzzy set theory tools and their potential application to a delivery hub location problem.

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