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MODIFICATION OF THE AHP METHOD BASED ON INTERVAL VALUED ROUGH NUMBERS

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Abstract

This paper presents a new approach for the treatment of uncertainty which is based on interval rough numbers (IRN). IRN based approach uses only internal knowledge i.e. operational data without relying on assumption models. In other words, instead of various additional/external parameters, IRN uses only the structure of the existing data. Based on the IRN there has been developed a novel IRAHP model (Interval Rough Analytic Hierarchy Process). The IR-AHP model has been tested on the case evaluation of outsourcing transport service. Modified IR-AHP method was used to determine the weights of coefficients of criteria in group decision-making process. The results of IR-AHP model were compared with the results given by the traditional AHP method and fuzzy AHP approach.

Key words*: rough numbers, group decision making, interval rough Analytic Hierarchy Process.*

1 INTRODUCTION

Collaborative decision making process is characterized by significant uncertainty and subjectivity of the decision makers who often have dilemma when assigning certain values to decision attributes. For the purpose of processing of uncertainty contained in data in the course of group decision making process, this paper deals with novel approach in rough sets theory, which is based on interval rough numbers (IRN). Suppose that a single decision attribute is assigned a value presented by qualitative scale with values ranging from 1 to 5. A single decision maker (DM) may consider that the attribute should have value between 3 and 4, the other one DM can believe that the attribute should be assigned a value between 4 and 5, while the third DM has no dilemma about the attribute value and decides to assign value 4. These dilemmas are very common in the group decision making process. In such cases, a compromise solution may be to find geometric mean of two values. However, this would result in lack of uncertainty in the decision making process which will further lead to crisp values. On the other side, application of fuzzy or grey techniques would imply presence of uncertainty and subjective definition of intervals for presenting uncertainty. Subjectivity in intervals, which is used to express uncertainty could significantly affect the final decision. Therefore subjectivity should be put aside to enable objective decision making process.

For treating uncertainty in multi-criteria decision making process, the majority of authors use fuzzy sets as their starting assumption [1] or various extensions of fuzzy theory. In addition to fuzzy theory, rough sets theory originally introduced by Pawlak [2] is another suitable tool for treating uncertainties. In recent years, rough sets theory has been successfully implemented in various fields of human activities [3-8].

The intention of interval fuzzy method in the decision making process is to enable transformation of crisp numbers into fuzzy numbers that imply uncertainties in real world systems by using membership function. Unlike the fuzzy sets theory whose application requires partial membership function without clearly defined set boundaries, the rough set theory uses limit set area for expressing uncertainties. While fuzzy and probability theories define degree of uncertainty based on assumption, the rough set theory deals with uncertainty based on approximation, which is the basic concept of the rough sets theory. Since the rough sets theory deals exclusively with internal knowledge and operational data there is no need for introduction of assumption models. In other words, when applying rough sets, only structure of the given data will be used instead of various additional/external parameters [9]. When dealing with rough sets, measurement of uncertainty is done based on vagueness, which is already contained in data [10]. In such a way, objective indicators contained in data can be determined.

Since this paper deals with application of interval rough numbers in multi-criteria decision making process, the text below will present studies referring to modification of multi-criteria decision making (MCDM) models by applying rough numbers and their extensions.

There are certain papers that deal with application of rough numbers with multi-criteria models that utilize rough AHP method either individually [3-5] or as hybrid model in combination with other multi-criteria techniques: AHP-TOPSIS [7], AHP-VIKOR [6] and AHP-MABAC [11]. However, the available literature has also shown lack of studies that deal with application of rough sets and rough numbers in MCDM process even though they showed considerable advantages. Regarding application of interval rough numbers in MCDM, there are only few papers dealing with this subject. Wang et al [12] applied interval rough numbers in dealing with uncertainty when determining weight coefficients of decision attributes by introducing interval rough operator for IRN aggregation.

IRN was also applied for development of hybrid QFD model [13].

This paper has several objectives. The first one refers to upgrading of the methodology for treating uncertainty in group MCDM process. The second objective is to affirm the IRN idea through detail presentation of arithmetic operations with IRN typical for MCDM. The third objective is to initiate the other authors to start with wide application of IRN in MCDM, since advantages of IRN, as presented in this paper, shows reasonable motive for widespread application. And finally, the forth objective of this paper is to bridge the gap identified in the methodology for bidder evaluation process in public procurement procedure by applying novel approach in treating uncertainties based on IRN.

The remainder of this paper is structured as follows: the second section of the paper shows the basic settings of IRN, arithmetic operations with IRN and the matching algorithm for the comparison of IRN. The third section of the paper presents a novel IR-AHP model in stages of the application. In the fourth section, we present the application of IR-AHP model in the evaluation of outsourcing transport service.

2 INTERVAL ROUGH NUMBERS

Let us assume that there is a set of k classes that represent preferencies of DM, $R=(J_1,J_2,...,J_k)$, with the condition that they belong to the string that satisfies the condition $J_1<J_2<,...,<J_k$ and another set of m classes that also represent preferencies of DM, $R^*=(I_1,I_2,...,I_k)$. All the objects are defined in hte universe and connected with the preferencies of DM. In R^* each class of objects is represented in the interval $I_i=\{I_{li},I_{ui}\}$, with the satisfied condition that $I_{li}\le I_{ui}$ ($1\le i\le m$), as well as the condition that $I_{li},I_{ui}\in R$. In this case I_{li} represents the lower border of the interval, while I_{ui} represents the upper limit of the interval of the *i* class of objects. If both limits of the classes of objects (upper and lower limit) are set in such a way that $I_{l1}^*<I_{l2}^*<,...,<I_{lj}^*,I_{u1}^*<I_{u2}^*<,...,<I_{uk}^*$ ($1\le j,k\le m$), respectively, then we can define two new sets that contain the lower class of the objects $R_l^*=(I_{l1}^*,I_{l2}^*,...,I_{lj}^*)$ and the upper class of the objects $R_u^*=(I_{u1}^*,I_{u2}^*,...,I_{uk}^*)$, respectively. In that case for any class of objects $I_{li}^*\in R$ ($1\le i\le j$) i $I_{ui}^*\in R$ ($1\le i\le k$) we can define the lower approximation of I_{li}^* and I_{ui}^* in the following way:

$$\underline{Apr}(I_{li}^*)=\cup\{Y\in U/R_l^*(Y)\le I_{li}^*\} \tag{1}$$

$$\underline{Apr}(I_{ui}^*)=\cup\{Y\in U/R_u^*(Y)\le I_{ui}^*\} \tag{2}$$

Upper approximation of I_{li}^* i I_{ui}^* is defined by the use of the following:

$$\overline{Apr}(I_{li}^*)=\cup\{Y\in U/R_l^*(Y)\ge I_{li}^*\} \tag{3}$$

$$\overline{Apr}(I_{ui}^*)=\cup\{Y\in U/R_u^*(Y)\ge I_{ui}^*\} \tag{4}$$

Both classes of objects (upper and lower class of objects I_{li}^* i I_{ui}^*) are defined by their lower limits $\underline{Lim}(I_{li}^*)$ i $\underline{Lim}(I_{ui}^*)$ and upper limits) $\overline{Lim}(I_{li}^*)$ i $\overline{Lim}(I_{ui}^*)$, respectively.

$$\underline{Lim}(I_{li}^*)=\frac{1}{M_L}\sum R_l^*(Y)\big|Y\in\underline{Apr}(I_{li}^*) \tag{5}$$

$$\underline{Lim}(I_{ui}^*)=\frac{1}{M_L^*}\sum R_u^*(Y)\big|Y\in\underline{Apr}(I_{ui}^*) \tag{6}$$

where M_L i M_L^* represent the sum of objects contained in the lower approximation of the classes of objects I_{li}^* i I_{ui}^*, respectively. Upper limits $\overline{Lim}(I_{li}^*)$ i $\overline{Lim}(I_{ui}^*)$ are defined by the following (7) i (8)

$$\overline{Lim}(I_{li}^*)=\frac{1}{M_U}\sum R_l^*(Y)\big|Y\in\overline{Apr}(I_{li}^*) \tag{7}$$

$$\overline{Lim}(I_{ui}^*)=\frac{1}{M_U^*}\sum R_u^*(Y)\big|Y\in\overline{Apr}(I_{ui}^*) \tag{8}$$

where M_U i M_U^* represent the sum of objects contained in the upper approximation of the classes of objects I_{li}^* i I_{ui}^*, respectively.

For the lower class of objects, the rough boundary interval) of I_{li}^* is represented as $RB(I_{li}^*)$ and it represent the interval between lower and upper limit:

$$RB(I_{li}^*)=\overline{Lim}(I_{li}^*)-\underline{Lim}(I_{li}^*) \tag{9}$$

For the upper class of objects, the rough boundary interval of I_{ui}^* we get as

$$RB(I_{ui}^*)=\overline{Lim}(I_{ui}^*)-\underline{Lim}(I_{ui}^*) \tag{10}$$

Then uncertain class of objects I_{li}^* i I_{ui}^* we can represent by their lower and upper limits

$$RN(I_{li}^*)=\left[\overline{Lim}(I_{li}^*),\underline{Lim}(I_{li}^*)\right] \tag{11}$$

$$RN(I_{ui}^*)=\left[\overline{Lim}(I_{ui}^*),\underline{Lim}(I_{ui}^*)\right] \tag{12}$$

As we can see, each class of objects is defined by its upper and lower limits that are made of interval rough numbers, which are defined as

$$IRN(I_i^*)=\left[RN(I_{li}^*),RN(I_{ui}^*)\right] \tag{13}$$

Interval rough numbers are characterized by specific arithmetic operations, which differ from the arithmetic operations with classic rough numbers. The next section defines the basic arithmetic operations with interval rough numbers.

Arithmetic operations between two rough interval numbers $IRN(A)=([a_1,a_2],[a_3,a_4])$ and $IRN(B)=([b_1,b_2],[b_3,b_4])$ are performed by the using of the following (14), (15), (16), (17) i (18):

(1) Adding of interval rough numbers "+"

$$IRN(A)+IRN(B)=([a_1,a_2],[a_3,a_4])+([b_1,b_2],[b_3,b_4])$$
$$=([a_1+b_1,a_2+b_2],[a_3+b_3,a_4+b_4])$$

(2) Revocation of interval rough numbers "-"

$$IRN(A)-IRN(B)=([a_1,a_2],[a_3,a_4])-([b_1,b_2],[b_3,b_4])$$
$$=([a_1-b_4,a_2-b_3],[a_3-b_2,a_4-b_1])$$

(3) Multiplication of interval rough numbers "×"

$$IRN(A)\times IRN(B)=([a_1,a_2],[a_3,a_4])\times([b_1,b_2],[b_3,b_4])$$
$$=([a_1\times b_1,a_2\times b_2],[a_3\times b_3,a_4\times b_4])$$

(4) Division of interval rough numbers "/"

$$IRN(A)/IRN(B)=([a_1,a_2],[a_3,a_4])/([b_1,b_2],[b_3,b_4])$$
$$=([a_1/b_4,a_2/b_3],[a_3/b_2,a_4/b_1])$$

(5) Scalar multiplication of interval rough numbers where $k > 0$

$$k \times IRN(A) = k \times ([a_1, a_2],[a_3, a_4])$$
$$= ([k \times a_1, k \times a_2],[k \times a_3, k \times a_4])$$

Any two interval rough numbers $IRN(\alpha) = ([\alpha^L, \alpha^U],[\alpha'^L, \alpha'^U])$ and $IRN(\beta) = ([\beta^L, \beta^U],[\beta'^L, \beta'^U])$ are ranked by the use of the following rules (Figure 1):

(1) In an interval of the interval rough number is not strictly bounded by other interval, then:

(a) If the condition is satisfied that { $\alpha'^U > \beta'^U$ i $\alpha^L \geq \beta^L$ } or { $\alpha'^U \geq \beta'^U$ i $\alpha^L < \beta^L$ } then $IRN(\alpha) > IRN(\beta)$, Figure 1a.

(b) If the condition is satisfied that { $\alpha'^U = \beta'^U$ i $\alpha^L = \beta^L$ } then je $IRN(\alpha) = IRN(\beta)$, Figure 1b.

(2) If the intervals of interval rough numbers $IRN(\alpha)$ and $IRN(\beta)$ are strictly bounded, then it is necessary to determine the points of intersection $I(\alpha)$ i $I(\beta)$ of the intervaln rough numbers $IRN(\alpha)$ and $IRN(\beta)$. Then, if the condition is satisfied that $\beta'^U < \alpha'^U$ and $\beta^L > \alpha^L$

(a) If the condition is satisfied that $I(\alpha) \leq I(\beta)$ then $IRN(\alpha) < IRN(\beta)$, Figure 1c and 1d.

(c) If the condition is satisfied that $I(\alpha) > I(\beta)$ then $IRN(\alpha) > IRN(\beta)$, Figure 1e.

We get the intersection points in the following way

$$\mu_\alpha = \frac{RB(\alpha_{ui})}{RB(\alpha_{ui}) + RB(\alpha_{li})}; \quad RB(\alpha_{ui}) = \alpha'^U - \alpha'^L; \; RB(\alpha_{li}) = \alpha^U - \alpha^L \tag{14}$$

$$\mu_\beta = \frac{RB(\beta_{ui})}{RB(\beta_{ui}) + RB(\beta_{li})}; \quad RB(\beta_{ui}) = \beta'^U - \beta'^L; \; RB(\beta_{li}) = \beta^U - \beta^L \tag{15}$$

$$I(\alpha) = \mu_\alpha \cdot \alpha^L + (1 - \mu_\alpha) \cdot \alpha'^U \tag{16}$$

$$I(\beta) = \mu_\beta \cdot \beta^L + (1 - \mu_\beta) \cdot \beta'^U \tag{17}$$

Similar rules may be performed in the case that $\alpha'^U < \beta'^U$ i $\alpha^L > \beta^L$.

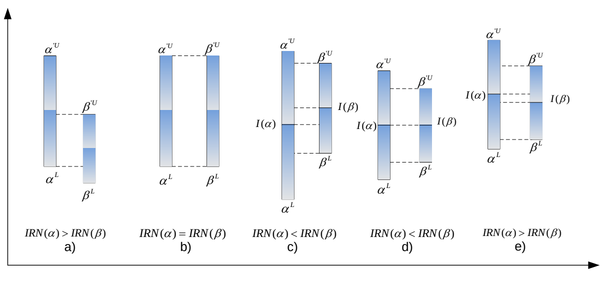

Figure 1. Ranking of interval rough numbers

3 IR-AHP method

This paper present the novel approach in the implementation of IRN in the group decision-making that has been presented as IR-AHP model. Interval rough numbers have been used to exploit ambiguity in group decision-making. IR-AHP method was used to determine the weights coefficient criteria.

Step 1. The formation of the hierarchical structure of evaluation criteria. We form a group of e experts who select the criteria and define the hierarchy of the problem with the global aim on top and criteria on the lower level.

Step 2. Filling the matrix for comparison in pairs of evaluation criteria. The members of the group of experts perform comparison in pairs of evaluation criteria with the aim of defining the weight coefficient of criteria. Comparison in pairs is performed by *Saaty's* 9-level linguistic scale [14]. Every *e* expert presents his comparisons with the following matrix

$$Z_k = \begin{bmatrix} 1 & z_{12}^e; z_{12}^{e'} & \cdots & z_{1n}^e; z_{1n}^{e'} \\ z_{21}^e; z_{21}^{e'} & 1 & \cdots & z_{2n}^e; z_{2n}^{e'} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}^e; z_{n1}^{e'} & z_{n2}^e; z_{n2}^{e'} & \cdots & 1 \end{bmatrix}_{nxn} \tag{18}$$

where $1 \leq i, j \leq n$; $1 \leq k \leq e$, z_{ij}^e and $z_{ij}^{e'}$ are linguistic expressions of *Saaty's* 9-level linguistic scale where *e* expert presents his comparision in pairs of criteria.

In case that *e* expert has uncertainty with comparing pairs of criteria (i, j), i.e. e expert cannot decide between two values of *Saaty's* 9-level linguistic scale, then both values from the scale are given ($z_{ij}^e \neq z_{ij}^{e'}$). In case there is no uncertainty, then *e* expert without doubt chooses one value. Then into the matrix of comparison of criteria (Z_k) we enter the same value on the position (i, j), i.e. $z_{ij}^e = z_{ij}^{e'}$. For example, during the comparison of criteria at the position (1,2) an expert cannot decide between two linguistical values (e.g. 5 i 6), then on the position (1,2) in the matrix Z_e we enter $z_{12}^e = 5$, i.e. $z_{12}^{e'} = 6$.

This way we get Z_1, Z_2,..., Z_e of matrix in which *e* experts have performed comparisons in the pairs of criteria.

Step 3. Determination of weight coefficients of experts. For each comparison matrix Z_k, we determine consistency of value of experts. In order to check consistency Saaty [15] has proposed *consistency ratio – CR*. Calculation of the degree of consistency takes place in two steps. In the first step we determine the *consistency index – CI* $CI = (\lambda_{\max} - n)/(n-1)$, where n is the matrix rank, and $\lambda_{\max}$ is the maximum own value of the comparison matrix.

In the second step the consistency ratio (CR) is calculated as a relation between consistency index and random index (RI)

$$CR = \frac{CI}{RI} \tag{19}$$

Random index (RI) depends on the rank of the matrix and its values are determined by randomly generating 500 matrix [15]. If the consistency ratio (*CR*) is less or equals 0.10 the result means that the expert has been consistent and there is no need to repeat the evaluation [14]. If CR is more than 0.10 the decision maker should repeat (or modify) his evaluations in order to improve his consistency. Since every expert into the matrix Z_k at the position (*i,j*) enters two values, we get two consistency ratios for each expert, i.e. CR^e i $CR^{e'}$. The final degree of consistency of the expert is the middle value of performed consistency ratios $CR_e = (CR^e + CR^{e'})/2$.

Expert's weight coefficients are determined by the normalization of reciprocal values of consistency ration, expression (20) i (21).

$$W_{ke} = \frac{1}{CR_e} \tag{20}$$

where CR_e is the consistency ratio of the expert, *e*, and W_{ke} is the weight coefficient of the expert *e*.
Normalization of the weight coefficients is performed by adding normalization

$$w_{ke} = \frac{W_{ke}}{\sum_{k=1}^{e} W_{ke}} \tag{21}$$

where W_{ke} is the weight coefficient of the expert *e*.

Step 4. Constructing of averaged interval rough comparison matrix. By applying expressions (1)-(13) the elements z_{ij}^{e} and $z_{ij}^{e'}$ of the comparison matrix Z_k are transformed into interval rough number $IRN(z_{ij}^{e})$

$$IRN(z_{ij}^{e}) = \left[RN(z_{ij}^{eL}), RN(z_{ij}^{e'U})\right] \tag{22}$$

where z_{ij}^{eL} and $z_{ij}^{e'L}$ represent the lower approximation of the object class z_{ij}^{e} and $z_{ij}^{e'}$, respectively, while z_{ij}^{eU} and $z_{ij}^{e'U}$ respectively represent the upper approximation of the object class z_{ij}^{e} and $z_{ij}^{e'}$.

This way for each comparison matrix in pairs by the expert *e* we get rough sequences $RN(z_{ij}^{eL})$ and $RN(z_{ij}^{e'U})$ which we present by the expression (28) i (29)

$$RN(z_{ij}^{eL}) = \left\{\left[z_{ij}^{1L}, z_{ij}^{1U}\right], \left[z_{ij}^{2L}, z_{ij}^{2U}\right], ..., \left[z_{ij}^{eL}, z_{ij}^{eU}\right]\right\} \tag{23}$$

$$RN(z_{ij}^{e'U}) = \left\{\left[z_{ij}^{1'L}, z_{ij}^{1'U}\right], \left[z_{ij}^{2'L}, z_{ij}^{2'U}\right], ..., \left[z_{ij}^{e'L}, z_{ij}^{e'U}\right]\right\} \tag{24}$$

As shown in the expression (27) rough sequences (28) and (29) make up the interval rough number $IRN(z_{ij}^{e})$ For each matrix Z_k we get two rough sequences which make up the interval rough number. By applying expressions (27) and (28) we get the averaged interval rough number $IRN(z_{ij})$

$$IRN(z_{ij}) = \left[RN(z_{ij}^{L}), RN(z_{ij}^{'U})\right] = \left(\left[z_{ij}^{L}, z_{ij}^{U}\right], \left[z_{ij}^{'L}, z_{ij}^{'U}\right]\right) \tag{25}$$

$$RN(z_{ij}^{L}) = RN(z_{ij}^{1L}, z_{ij}^{2L}, ..., z_{ij}^{eL}) = \begin{cases} z_{ij}^{L} = \prod_{k=1}^{e} z_{ij}^{eL(w_{ke})} \\ z_{ij}^{U} = \prod_{k=1}^{e} z_{ij}^{eU(w_{ke})} \end{cases} \tag{26}$$

$$RN(z_{ij}^{'U}) = RN(z_{ij}^{1'L}, z_{ij}^{2'L}, ..., z_{ij}^{e'L} = \begin{cases} z_{ij}^{'L} = \prod_{k=1}^{e} z_{ij}^{e'L(w_{ke})} \\ z_{ij}^{'U} = \prod_{k=1}^{e} z_{ij}^{e'U(w_{ke})} \end{cases} \tag{27}$$

where w_k is the weights coefficient of the *k* expert ($k = 1, 2, ..., e$), $RN(z_{ij}^{L})$ i $RN(z_{ij}^{'U})$ represent the lower and upper limit of the interval rough number $IRN(z_{ij})$, respectively.

$$Z = \begin{bmatrix} 1 & IRN(z_{12}) & \cdots & IRN(z_{1n}) \\ IRN(z_{21}) & 1 & \cdots & IRN(z_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ IRN(z_{n1}) & IRN(z_{n2}) & \cdots & 1 \end{bmatrix}_{nxn} \tag{28}$$

Based on the expression (25) matrix (28) we can also present as

$$Z = \begin{bmatrix} 1 & \left(\left[z_{12}^{L}, z_{12}^{U}\right], \left[z_{12}^{'L}, z_{12}^{'U}\right]\right) & \text{L} & \left(\left[z_{1n}^{L}, z_{1n}^{U}\right], \left[z_{1n}^{'L}, z_{1n}^{'U}\right]\right) \\ \left(\left[z_{21}^{L}, z_{21}^{U}\right], \left[z_{21}^{'L}, z_{21}^{'U}\right]\right) & 1 & \text{L} & \left(\left[z_{2n}^{L}, z_{2n}^{U}\right], \left[z_{2n}^{'L}, z_{2n}^{'U}\right]\right) \\ \text{M} & \text{M} & \text{O} & \text{M} \\ \left(\left[z_{n1}^{L}, z_{n1}^{U}\right], \left[z_{n1}^{'L}, z_{n1}^{'U}\right]\right) & \left(\left[z_{n2}^{L}, z_{n2}^{U}\right], \left[z_{n2}^{'L}, z_{n2}^{'U}\right]\right) & \text{L} & 1 \end{bmatrix}_{nxn} \tag{29}$$

Step 5. Calculation of vector priority criteria. Priority vector is the interval rough weights coefficient $IRN(w_j)$ and it is determined for each *n* of the evaluation criteria. Interval rough weights coefficient $IRN(w_j)$ we get by appalying the expressions (30)-(33). By applying expression (30) elements of the matrix *Z* are summed up by the columns

$$IRN(a_{ij}^{'}) = \sum_{j=1}^{n} IRN(z_{ij}) = \left(\left[\sum_{j=1}^{n} z_{ij}^{L}, \sum_{j=1}^{n} z_{ij}^{U}\right], \left[\sum_{j=1}^{n} z_{ij}^{'L}, \sum_{j=1}^{n} z_{ij}^{'U}\right]\right) \tag{30}$$

By dividing the matrix elements (29) with the values from the expression (30) we get a normalized matrix of weights coefficient *W*, expressions (31) and (32).

$$IRN(w_{ij}) = \left(\left[w_{ij}^{L}, w_{ij}^{U}\right], \left[w_{ij}^{'L}, w_{ij}^{'U}\right]\right) = \frac{IRN(z_{ij})}{\sum_{j=1}^{n} IRN(z_{ij})} = \frac{\left(\left[z_{ij}^{L}, z_{ij}^{U}\right], \left[z_{ij}^{'L}, z_{ij}^{'U}\right]\right)}{\left(\left[\sum_{j=1}^{n} z_{ij}^{L}, \sum_{j=1}^{n} z_{ij}^{U}\right], \left[\sum_{j=1}^{n} z_{ij}^{'L}, \sum_{j=1}^{n} z_{ij}^{'U}\right]\right)} \tag{31}$$

$$W = \begin{bmatrix} 1 & \left(\left[w_{12}^{L}, w_{12}^{U}\right], \left[w_{12}^{'L}, w_{12}^{'U}\right]\right) & \text{L} & \left(\left[w_{1n}^{L}, w_{1n}^{U}\right], \left[w_{1n}^{'L}, w_{1n}^{'U}\right]\right) \\ \left(\left[w_{21}^{L}, w_{21}^{U}\right], \left[w_{21}^{'L}, w_{21}^{'U}\right]\right) & 1 & \text{L} & \left(\left[w_{2n}^{L}, w_{2n}^{U}\right], \left[w_{2n}^{'L}, w_{2n}^{'U}\right]\right) \\ \text{M} & \text{M} & \text{O} & \text{M} \\ \left(\left[w_{n1}^{L}, w_{n1}^{U}\right], \left[w_{n1}^{'L}, w_{n1}^{'U}\right]\right) & \left(\left[w_{n2}^{L}, w_{n2}^{U}\right], \left[w_{n2}^{'L}, w_{n2}^{'U}\right]\right) & \text{L} & 1 \end{bmatrix}_{nxn} \tag{32}$$

Final rough interval weights coefficients ($IRN(w_j)$) of the evaluation criteria are determined by the expression (33)

$$IRN(w_j)=\left(\left[\sum_{i=1}^{n}w_{ij}^{L},\sum_{i=1}^{n}w_{ij}^{U}\right],\left[\sum_{i=1}^{n}w_{ij}^{'L},\sum_{i=1}^{n}w_{ij}^{'U}\right]\right)\Big/n \qquad (33)$$

where *n* represents the number of evaluation criteria.
The value of weights coefficient criteria are placed in the interval $IRN(w_j)=\left(\left[w_j^L,w_j^U\right],\left[w_j^{'L},w_j^{'U}\right]\right)$ where the condition is satisfied that $0\le w_j^L\le w_j^{'L}\le w_j^U\le w_j^{'U}\le 1$ for each evaluation criteria e $x_j\in X$. However, it is necessary that a condition is satisfied that generally the sum of weights coefficient criteria equals one. In our case, since these are interval rough weights coefficient critera, by using the expression (33) we get weights coefficients where $\sum_{j=1}^{n}w_j^L\le 1$, $\sum_{j=1}^{n}w_j^{'L}\le 1$, $\sum_{j=1}^{n}w_j^U\ge 1$ i $\sum_{j=1}^{n}w_j^{'U}\ge 1$.
This satisfies the condition that weights coefficient must be in the interval $w_i\in[0,1],\quad (i=1,2,...,n)$.

4 Application of the IR-AHP method

IR-AHP method was tested in the case of transport providers selection. A group of providers was examined, whose names are not listed due to the protection of business data. Four transportation experts participated in the process of model testing. The chosen experts have had minimum five years of experience in outsourcing of transportation services and in provision of transportation consulting services. In the interviewing process, the data about criteria interdependence were collected, and afterwards the experts' opinions aggregation was performed.
From a wide range of criteria, for the carrier selection in this paper we have used: Total cost (C1), Reliability (C2), Business excellence (C3), Customer service (C4) and Green image (C5).
The first phase of implementation of IR-AHP method means comparison in pairs of evaluation critera by the experts that participate in this research. After expert evaluation of each criteria for every single expert, we got a matrix of comparison of criteria in pairs Table 1.
That means that the expert could not with certainty choose one of the values, either 4 or 5. In case there is no uncertainty, than expert *e* without doubt chooses one value. Such an example is position C2-C5 in matrix DM1. The first expert DM1 into the comparison matrix puts two same values, i.e. (z_{25}=9; z'_{25}=9).
After comparison in pairs of criteria we then determine the ratio of consistency of comparison matrix. Since an expert for each position in matrix Z_k enters two values ($z_{ij}^{e};z_{ij}^{e'}$), we get two ratios of consistency for each expert, i.e. CR^e i $CR^{e'}$.

Table 1. Matrix of comparison in pairs of evaluation criteria

			E1		
	C1	C2	C3	C4	C5
C_1	(1.00;1.00)	(3.00;5.00)	(4.00;5.00)	(0.33;0.25)	(6.00;7.00)
C_2	(0.33;0.20)	(1.00;1.00)	(1.00;1.00)	(0.17;0.14)	(3.00;3.00)
C_3	(0.25;0.20)	(1.00;1.00)	(1.00;1.00)	(0.17;0.14)	(2.00;3.00)
C_4	(3.00;4.00)	(6.00;7.00)	(6.00;7.00)	(1.00;1.00)	(8.00;9.00)
C_5	(0.17;0.14)	(0.33;0.33)	(0.50;0.33)	(0.11;0.13)	(1.00;1.00)
			…		
			E4		
	C1	C2	C3	C4	C5
C_1	(1.00;1.00)	(4.00;5.00)	(4.00;5.00)	(0.14;0.13)	(2.00;3.00)
C_2	(0.25;0.20)	(1.00;1.00)	(1.00;1.00)	(0.11;0.13)	(0.50;0.33)
C_3	(0.25;0.20)	(1.00;1.00)	(1.00;1.00)	(0.11;0.11)	(0.33;0.25)
C_4	(7.00;8.00)	(8.00;9.00)	(9.00;9.00)	(1.00;1.00)	(7.00;8.00)
C_5	(0.50;0.33)	(2.00;3.00)	(3.00;4.00)	(0.14;0.13)	(1.00;1.00)

After the calculation of the consistency ratio of the comparison matrix (Table 2) we can conclude that the research is valid because for all the values CR_e<0.1. The final consistency ration of the for example first expert (Table 2) we get as the average value of the determined consistency ratios CR_e=(0.062+0.083)/2=0.072.

Table 2. CR_e matrix of comparison and weights of experts

Ekspert	CR^e	$CR^{e'}$	CR_e	w_{ke}
E 1	0.083	0.095	0.089	0.200
E 2	0.046	0.099	0.072	0.247
E 3	0.036	0.093	0.065	0.274
E 4	0.044	0.085	0.064	0.278

In order to get averaged interval rough matrix of comparison, based on the data from Table 1 and by using the experession (1)-(13), the elements z_{ij}^{e} i $z_{ij}^{e'}$ of the comparison matrix Z_k are transformed into the interval rough number $IRN(z_{ij}^{e})$. This way we get nine interval rough matrix Z_k.
Determination of interval rough elements of comparison matrix Z_1, Z_2,..., Z_9, we shall show on the example of the elements at the position C2-C4.
Interval rough number (22) consists of two rough sequences (23) and (24). For every matrix Z_k we get two rough sequences which make up the interval rough number (22).
From the comparison matrix (Table 3), for the position C2-

C4 we select to object classes z_{ij}^{e} and $z_{ij}^{e'}$. Each class contains nine elements:

$$z_{24}^{e} = \{0.17;\ 2;\ 3;\ 0.11\}$$

$$z_{24}^{e'} = \{0.14;\ 3;\ 4;\ 0.125\}$$

By applying the expression (1)-(8) we form rough sequences for each objects class. For the first object class we get:

$$\underline{Lim}(0.17) = \frac{1}{2}(0.17 + 0.11) = 0.14\,,$$

$$\overline{Lim}(0.17) = \frac{1}{3}(0.17 + 2 + 3) = 1.72\,;$$

$$\underline{Lim}(2) = \frac{1}{3}(0.17 + 2 + 0.11) = 0.76\,,\ \overline{Lim}(2) = \frac{1}{2}(2 + 3) = 2.5\,;$$

...

$$\underline{Lim}(0.11) = 0.11\,,\ \overline{Lim}(0.11) = \frac{1}{4}(0.17 + 2 + 3 + 0.11) = 1.32\,;$$

For the second object class we get:

$$\underline{Lim}(0.14) = \frac{1}{2}(0.14 + 0.125) = 0.13\,,$$

$$\overline{Lim}(0.14) = \frac{1}{3}(0.14 + 3 + 4) = 2.38\,;$$

$$\underline{Lim}(3) = \frac{1}{3}(0.14 + 3 + 0.125) = 1.08\,,\ \overline{Lim}(3) = \frac{1}{2}(3 + 4) = 3.5\,;$$

...

$$\underline{Lim}(0.125) = 0.125\,,$$

$$\overline{Lim}(0.125) = \frac{1}{4}(0.14 + 3 + 4 + 0.125) = 1.82\,.$$

This way we get rough sequences that make up the interval rough number:

$$RN(z_{24}^{1L}) = [0.114,1.72];\ \ RN(z_{24}^{1'U}) = [0.13,2.38]\ \rightarrow\ IRN(z_{24}^{1}) = \left([0.14,1.72],[0.13,2.38]\right);$$

$$RN(z_{24}^{2L}) = [0.76,2.50];\ \ RN(z_{24}^{2'U}) = [1.08,3.50]\ \rightarrow\ IRN(z_{24}^{2}) = \left([0.76,2.50],[1.08,3.50]\right);$$

...

$$RN(z_{24}^{4L}) = [0.11,1.32];\ \ RN(z_{24}^{4'U}) = [0.125,1.82]\ \rightarrow\ IRN(z_{24}^{4}) = \left([0.11,1.32],[0.125,1.82]\right)$$

By applying the expression (26), (27) and weights coefficients of experts (Table 2) we get the average interval rough number $IRN(z_{24}) = \left([0.27,1.89],[0.28,2.27]\right)$. This way we get the average interval rough comparison matrix in the pairs of evaluation criteria, Table 3. Based on the data from table 3, by implementing the expressions (30) and (31) we get the normalized matrix of weights coefficients *W*, table 4.

Table 3. Averaged matrix

	C1	C2	C3	C4	C5
C1	([1.79,4.95],[1.59,4.63])	([0.15,0.22],[0.16,0.22])	([0.59,4.41],[0.50,3.69])	([0.34,2.63],[0.37,2.60])	([0.17,0.70],[0.18,0.58])
C2	([2.46,6.19],[2.04,6.43])	([0.27,1.89],[0.28,2.27])	([0.72,5.73],[0.71,6.00])	([0.66,3.01],[0.86,3.71])	([0.28,2.48],[0.36,2.77])
C3	([1.00,1.00],[1.00,1.00])	([0.12,0.16],[0.13,0.19])	([0.31,2.06],[0.27,2.29])	([0.19,0.59],[0.19,0.92])	([0.13,0.23],[0.14,0.29])
C4	([6.36,8.31],[5.51,7.62])	([1.00,1.00],[1.00,1.00])	([3.16,7.00],[3.82,7.13])	([1.23,6.30],[1.37,6.47])	([0.76,2.94],[1.17,3.04])
C5	([1.57,4.45],[1.42,4.86])	([0.16,0.43],[0.15,0.32])	([1.00,1.00],[1.00,1.00])	([0.41,4.01],[0.37,4.01])	([0.20,2.01],[0.23,2.10])

Table 4. Normalized matrix of weights coefficients

	C1	C2	C3	C4	C5
C1	([0.05,0.24],[0.04,0.26])	([0.02,0.09],[0.02,0.09])	([0.02,0.53],[0.02,0.45])	([0.01,0.51],[0.02,0.51])	([0.01,0.21],[0.02,0.2])
C2	([0.06,0.3],[0.05,0.36])	([0.04,0.74],[0.04,0.94])	([0.02,0.69],[0.02,0.74])	([0.03,0.58],[0.04,0.73])	([0.02,0.74],[0.03,0.98])
C3	([0.03,0.05],[0.03,0.06])	([0.02,0.06],[0.02,0.08])	([0.01,0.25],[0.01,0.28])	([0.01,0.11],[0.01,0.18])	([0.01,0.07],[0.01,0.1])
C4	([0.16,0.4],[0.14,0.43])	([0.14,0.39],[0.14,0.42])	([0.09,0.85],[0.12,0.88])	([0.05,1.22],[0.06,1.27])	([0.06,0.88],[0.1,1.07])
C5	([0.04,0.22],[0.04,0.27])	([0.02,0.17],[0.02,0.13])	([0.03,0.12],[0.03,0.12])	([0.02,0.78],[0.02,0.79])	([0.02,0.6],[0.02,0.74])

By applying the expression (33) we get the interval rough weights coefficients of the evaluation criteria, Table 5.

In this paper we also determined weights coefficients by applying the traditional crisp AHP and fuzzy AHP (FAHP) methods. During the calculation of the values of the weights coefficients by FAHP method we used the symmetrical form of triangle fuzzy numbers. All three methods (AHP, F-AHP and IR-AHP) generate sequences of weights coefficient of the same rank (C4>C2> C5>C1>C3), but of different values.

Table 5. Weights coefficients of the evaluation criteria

Criteria	Interval rough approach	
	IRN(w_j)	Rank
C1	([0.02,0.30],[0.02,0.33])	4
C2	([0.03,0.53],[0.04,0.64])	2
C3	([0.01,0.11],[0.01,0.14])	5
C4	([0.10,0.80],[0.11,0.86])	1
C5	([0.02,0.42],[0.02,0.46])	3

Traditional crisp AHP method calculates the weight coefficients using crisp numbers. At the same time it does not take into account the uncertainties and ambiguities that exist in group decision-making process. In contrast, the fuzzy AHP and IR-AHP are using the weights of interval numbers with different size of the interval. Different sizes of the interval are the result of different mechanisms for the treatment of uncertainty and subjectivity. While fuzzy AHP model treats uncertainty by using fuzzy sets that have fixed limits depending on the membership function, in the interval rough numbers the interval boundaries are flexible and adapt to the prevailing uncertainties in the data. However, pre-defined limits with the FAHP interval further increase the subjectivity that rule in group decision-making. This may to a considerable extent further affect the degree of uncertainty that is expressed in size of the interval, i.e. the size of the interval uncertainty, which is not the case with IR-AHP. Thus, the proposed IR-AHP model can effectively measure ambiguities in the evaluation process of criteria and more objectively reflect the perceptions of decision-makers.

5 CONCLUSION

The basic idea behind the implementation of algorithms in decision making that are based on interval approach involves the use of interval numbers for presenting the value of attribute decisions. However, the boundaries of the interval in interval numbers are very difficult to determine and they are based on experience and intuition of decision-makers. In this paper we have used the novel approach for the exploitation of uncertainty in the decision making process which is based on interval rough numbers. The paper describes the basic concept of IRN, arithmetic operations and algorithms for their mutual comparison.

The approach based on IRN has been tested through the use of IR-AHP model in the case of evaluation of outsourcing transport service. The IR-AHP method was used to determine the weights of interval evaluation criteria.

Presented interval rough approach is applicable in other areas of MCDM. In future applications of IRN it would be interesting to show their use in combination with different models to determine weights of criteria. Further integration of interval rough approach in combination with the already existing models for decision making would facilitate to a great extent the exploitation of uncertainty and subjectivity prevailing in the decision making process, especially in models of group decision making.

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