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DISTANCE-CONSTRAINED CAPACITATED VEHICLE ROUTING PROBLEMS: CASE STUDY AND SIMULATED DATA SET

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Abstract

The distance-constrained capacitated vehicle routing problem (DCCVRP) is a combinatorial optimization problem. The DCCVRP looks at finding vehicle routes to connect all customers with a depot, such that the total distance is minimized; each customer is visited once by one vehicle; every route starts and ends at a depot; and the travelled distance and capacity by each vehicle is less than or equal to the given maximum value. This paper presents a heuristic and meta-heuristic approach for solving the DCCVRP and discusses the real problem of municipal waste in the city of Niš.

Key words*: vehicle routing problem, heuristic, meta-heuristic, waste collection.*

1 INTRODUCTION

The problems that are related to determining the routes that vehicles should take are very often encountered today. Waste and mail collection, newspaper, milk or bread distribution, post office packets delivery, assigning buses and planes to a network of lines, all of these represent problems that researchers and traffic experts face daily in practice [1]. The vehicle routing problem (VRP) is one of the most challenging problems of combinatorial optimization. This problem was first mentioned in 1959 by Dantzig and Ramser [2]. Since then, the VRP has been increasingly applied to solve various problems, and it has had a great economic significance for the reduction of operational costs in distribution systems.

With the aim of resolving real problems for the solution of the VRP, several limitations are usually included in the problem solution, such as a greater number of depots, different types of vehicles (homogeneous and heterogeneous), different types of customer demands (deterministic and stochastic), infrastructural limitations (one-way streets, forbidden roads), various types of performed services (collection, delivery and mixed type), etc. If all of these limitations are taken into account, then the VRP becomes much more complicated to solve and falls under the category of *NP* difficult problems. For that reason, the literature includes a variety of VRPs. The basic model of vehicle routing problems is the VRP with limited capacity or the capacitated vehicle routing problem (CVRP). The CVRP includes deterministic customer demands, which are known in advance and which cannot be divided. Vehicles are identical and have a common starting point, while the vehicle capacity presents the only limitation. The goal function expresses the demand to minimize the total costs. The CVRP is the basis from which various vehicle routing problems stem [3]. For the sake of simplicity, figure 1 uses abbreviations. The detailed explanation of figure 1 is given in the following text: DCCVPR – distance–constrained capacitated vehicle routing problem; VRPB – vehicle routing problem with backhauls; VRPTW – vehicle routing problem with time window; VRPPD – vehicle routing problem with pickup and delivery; VRPBTW – vehicle routing problem with backhauls and time window; VRPPDTW – vehicle routing problem with pickup and delivery and time window.

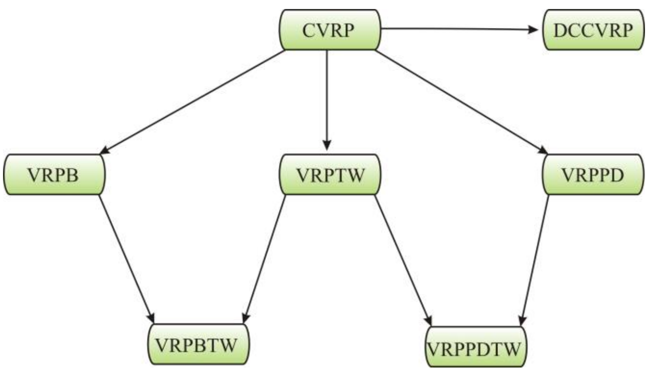

Fig. 1 *Basic vehicle routing problems and their relations*

Heuristic and meta-heuristic methods are most often used to solve the DCCVRP. This paper presents a model for municipal waste collection in the city of Niš on the basis of the DCCVRP. Several heuristic and meta-heuristic methods will be applied in order to simulate the model.

2 DISTANCE-CONSTRAINED CAPACITATED VEHICLE ROUTING PROBLEMS

As already mentioned, the basic vehicle routing problem is the VRP with limited capacity. When the capacity limitation is substituted by the maximal distance or time duration limitation, the VRP changes into the form called the distance-constrained vehicle routing problem. The variant in which two limitations are present: the limitation of the capacity and the limitation of the maximal route distance, is called the distance-constrained capacitated vehicle routing problem – DCCVRP.

2.1 Mathematical formulation of the DCCVRP for municipal waste collection

The DCCVRP for municipal waste collection (DCCVRP-MWC) can be expressed by using the basic symbols from the theory of graphs in the following manner. Let a full graph be defined by the following equation (1):

$$G = (V, A, D, T, Q, S) \tag{1}$$

where: $V = \{1,...,n\}$ is the set of nodes, A is the set of arcs, $D = [d_{ij}]$ is the matrix of the shortest distances for A, $T = [t_{ij}]$ is the matrix of the travelling time in arc A without the location service time, $Q = (q_1, q_2, ...,q_n)$ is the demand in nodes V, and $S = (s_1, s_2, ...,s_n)$ are the times spent in nodes V.

In this paper nodes $i = (2, ... , n)$ represent the locations of waste containers N where the waste generated by a certain amount of users is disposed of. The node with the index 1 represents the depot, i.e. the landfill from which the vehicles begin their operation. The distance matrix d_{ij} is connected with the node $(i,j) \in A$ and it represents the shortest possible distance between the node i and the node j. If $d_{ij} = d_{ji}$ for every $(i,j) \in$ A, then the problem is called the symmetrical DCCVRP-MWC. In the opposite case, we have the asymmetrical DCCVRP-MWC. This paper deals with a symmetrical DCCVRP-MWC. For the matrix of shortest distances D and the matrix of the movement time in branch T, the following inequalities are valid (2 and 3, respectively):

$$\begin{aligned} & d_{ik} + d_{kj} > d_{ij} \\ & \forall i, j, k \in V, h \neq i \neq j \end{aligned} \tag{2}$$

$$\begin{aligned} & t_{ik} + t_{kj} > t_{ij} \\ & \forall i, j, k \in V, h \neq i \neq j \end{aligned} \tag{3}$$

When the distance of the branch, i.e. the distance from the node i to the node j, is expressed in the travelling time T, then the service time related to each of the nodes is added to that time, and this service time represents the time that a vehicle spends in a node. The times spent in nodes (s_{ij}) are calculated by multiplying the number of containers (B_k) with the time used to empty a container (t_{pk}), i.e. using expression (4):

$$s_{ij} = B_k \cdot t_{pk_{ij}} \tag{4}$$

When the times spent in nodes are calculated, then one can calculate the travelling times (t_{ij}) using expression (5):

$$t_{ij} = t^{`}_{ij} + \frac{s_{ij}}{2} + \frac{s_{ji}}{2} \tag{5}$$

where t'_{ij} is the time of travelling from the node i to the node j without the time spent in the nodes themselves.

Solving the DCCVRP-MWC means determining the N routes (each route is connected to only one vehicle), where the total travelled distance in a route has to be minimal. The total travelled distance is obtained as the sum of the distances in each of the branches that belong to a route. In this paper, one vehicle collects all the waste from all of the locations. The solution should meet the following demands:

- each route has to start and end in the depot (landfill),
- each node participates in only one route,
- the sum of the volume of waste in the nodes serviced in one route must not be greater than the vehicle capacity K_{voz},
- the sum of the travelling time in a route must not be greater than the maximal allowed time spent on the route T_N.

3 ALGORITHMS FOR SOLVING THE DCCVRP FOR MUNICIPAL WASTE COLLECTION

Bearing in mind that the VRP belongs to NP – difficult problems [4], the application of exact methods often does not yield a solution in a reasonable period. The reason for this lies in the number of potential routes for the VRP rising very quickly, thus it is not realistic to expect that exact methods can generate solutions applicable in real time and demanded in practice. Contrary to exact methods, the application of heuristic and meta-heuristic methods does not represent the knowledge of the structure or the relations within the model of the problem than one needs to solve. It can be said that a heuristic technique is employed to find a good solution to a problem in a relatively short period, without any guarantees concerning its optimality, or even its closeness to an optimal solution. Heuristic methods are mainly based on route construction, where routes are constructed and improved iteratively with regard to the goal function [5]. Since this paper discusses the determination of the most favourable (optimal) routes for municipal waste collection, and also taking into account all of the above limitations, it can be assumed with great certainty that the sufficiently good solutions, determined by applying the global optimization methods, are very close to or fully optimal.Two heuristic methods are applied in the paper: the Clarke-Wright algorithm (C-W) and the 2-OPT algorithm [6, 7]. Meta-heuristic methods basically imitate natural processes such as the biological evolution, ants searching for food, bee behaviour, finding the best harmony, thermodynamic process of metal annealing, and similar processes [8]. The paper applies the meta-heuristic method called Simulated Annealing (SA).

4 CASE STUDY AND SIMULATED DATA SET

The problem considered in this paper is defined by a transport network which comprises a depot and 20 nodes (fig. 2). The nodes represent the locations of the containers for the disposal of municipal waste located on the territory of the municipality of Mediana, city of Niš. This transport network represents “Area 103” according to the division of the territory of the city of Niš by the PUC Mediana - Niš. The nodes (container locations) are defined by coordinates, i.e. geographic latitude and geographic longitude (table 1). Apart from the coordinates, table 1 also shows the number of containers in each node for the observed transport network. The locations that contain more than one containers are defined by a single node, i.e. one coordinate. The waste collection containers have the volume of 1.1 m^3. In the transport network, the initial and final node (the depot) is marked with “1”. The other nodes in the transport network are marked with numbers from 2 to 21. A waste collection vehicle starts from the depot, services all of the nodes in the transport network by visiting each node only once, and returns to the depot having completed its service. The depot in this case does not represent the landfill. The landfill is located outside of the city. The distance from the landfill to the depot is 9 km.

After the first node is serviced, the capacity of the vehicle is checked and the route duration time is measured. If the capacity is not full and the route duration time is not exceeded, another node is serviced. On the other hand, if one of the limitations is exceeded, the vehicle returns to the depot

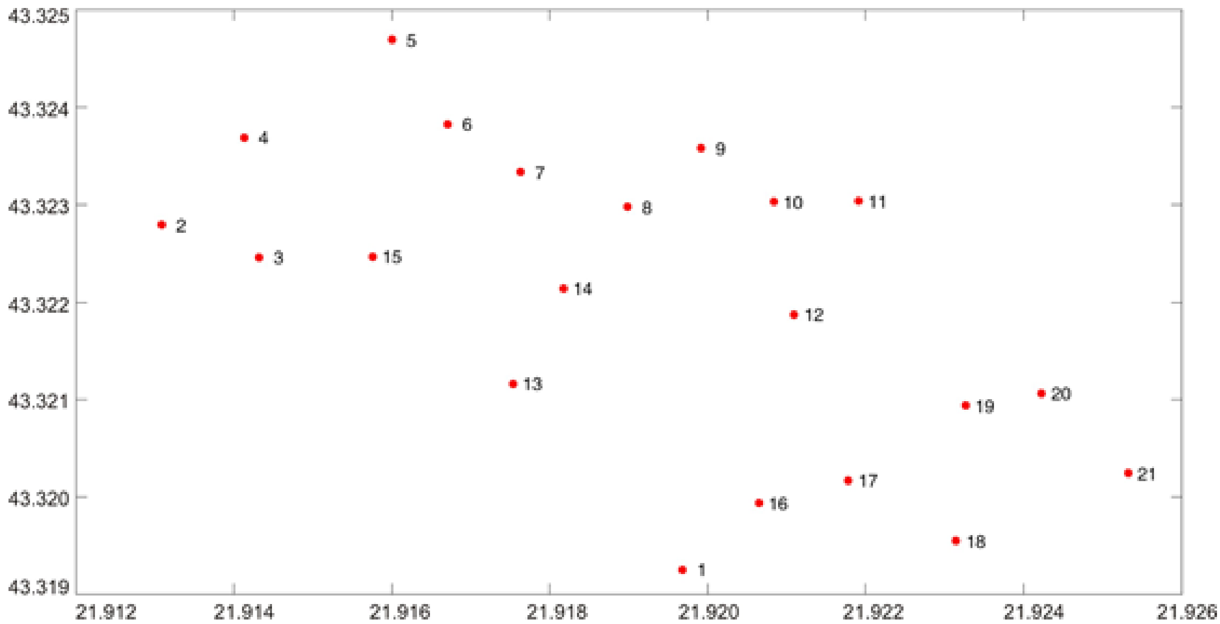

Fig. 2 *Transport network of nodes for municipal waste collection containers*

It is assumed that a vehicle leaving the depot is always empty. Before the vehicle leaves the depot, apart from its capacity (K_{voz}), what is also defined is the time limitation of the route duration (T_N).

Table 1. Coordinates and number of containers for the observed transport network

Ordinal number of location	Geographic latitude	Geographic longitude	Number of containers per location
Depot – 1	43.319256	21.919682	
2	43.322794	21.913082	18
3	43.322464	21.914317	17
4	43.324196	21.914412	11
5	43.324696	21.916001	8
6	43.323830	21.916709	14
7	43.323338	21.917632	21
8	43.322615	21.918220	4
9	43.323829	21.921069	9
10	43.322712	21.920759	8
11	43.322109	21.922545	4
12	43.321878	21.921097	4
13	43.321168	21.917535	15
14	43.322144	21.918179	11
15	43.322471	21.915755	12
16	43.319104	21.920206	5
17	43.319854	21.920548	8
18	43.320413	21.921150	7
19	43.320076	21.922198	11
20	43.320751	21.921739	8
21	43.320552	21.923959	8

to be emptied. This procedure is repeated until all of the nodes for the observed transport network have been serviced. During the service, it is not possible to perform a partial service that would imply meeting only a part of the demand in a specific node. This is the case of a closed vehicle routing system.

In this study the amount of waste per location is calculated as the number of containers multiplied by their volume. The matrix of demand per node is given in table 2. It is assumed that all of the containers are full to their maximum capacity, i.e. that the demand (the amount of waste) is known beforehand (deterministic).

Table 2. Amount of waste per node

Node	Demand [m^3]	Node	Demand [m^3]
2	19.8	12	4.4
3	18.7	13	16.5
4	12.1	14	12.1
5	8.8	15	13.2
6	15.4	16	5.5
7	23.1	17	8.8
8	4.4	18	7.7
9	9.9	19	12.1
10	8.8	20	8.8
11	4.4	21	8.8

One refuse vehicle with the capacity of 15 m^3 is provided for waste collection. The refuse vehicle (fig. 3) has a superstructure with a powerpress system. The degree of waste compaction is 4. This means that the refuse vehicle can collect the maximum of 60 m^3 of municipal waste, i.e. around 55 containers.

During the simulation of the DCCVRP-MCW, two limitations were taken into account: the first limitation is the vehicle capacity K_{voz} = 60 m^3, and the second one is the route duration time T_N = 60 min. The route duration time was set to 60 min because waste collection workers work for the total of 8 working hours (the duration of a single shift) with a 30-minute break. Apart from the limitation of the duration of all routes, there is also the time needed to drive the vehicle from the depot to the landfill, empty it there and return it to the depot.

The approximate time needed by the vehicle to travel from the depot to the landfill, empty the collected waste and return to the depot is around 50 min. Thus, when all these times are added together, they must not exceed the duration of a single shift.

Fig. 3 *Refuse vehicles with a powerpress system*

4.1 DCCVRP-MCW simulation results

The first DCCVRP-MCW simulation for the given transport network with 20 nodes was performed by applying the heuristic C-W algorithm. This simulation yielded four routes. The total duration time of the routes obtained by the simulation of DCCVRP-MCW using the C-W algorithm was 226.95 minutes. Thus obtained routes were improved by the 2-OPT local search with a varied number of iterations (1e3 and 1e5).

The improvement achieved by applying the 2-OPT local search in relation to the initial solution obtained by the C-W algorithm is shown in table 3. The routes obtained by the C-W algorithm and improved by the 2-OPT local search, as well as the individual route duration times and vehicle fullness, are shown in figure 3, while the layout of routes in the transport network is presented in figure 5.

The second DCCVRP-MCW simulation for the given transport network with 20 nodes was performed by applying the meta-heuristic SA algorithm. The SA parameters for which the simulation was performed are: initial temperature T_0 = 20, temperature reduction factor α = 0.98. The number of iterations was also varied. The limitation of the number of iterations was performed with 7e2 and 12e3. The SA algorithm application yielded four routes.

The total route duration time obtained by the DCCVRP-MCW simulation with the application of the SA algorithm is presented in table 3. The routes obtained by the SA algorithm, as well as the individual route duration times and vehicle fullness, are shown in figure 4, while the layout of routes in the transport network is presented in figure 6.

Table 3 Results of the local search with the initial C-W solution

Local search	Total route duration time	Improvement in relation to C-W [%]
2-OPT	226.95	-
2-OPT 1e3	222.01	2.18
2-OPT 1e5	222.01	2.18
SA 7e2	223.20	1.65
Sa 12e3	223.20	1.65

Рута 1
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 8 5 4 7
Укупно време трајања руте = 53.5
Укупна количина сакупљеног отпада у возилу = 48.40

Рута 2
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 14 3 15 17 16
Укупно време трајања руте = 60.00
Укупна количина сакупљеног отпада у возилу = 58.30

Рута 3
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 13 6 9 12 11 10
Укупно време трајања руте = 51.57
Укупна количина сакупљеног отпада у возилу = 59.4

Рута 4
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 19 2 21 20 18
Укупно време трајања руте = 56.94
Укупна количина сакупљеног отпада у возилу = 57.20

Fig 3 *Route generation on the basis of the limitations of vehicle capacity and route duration time by applying the C-W algorithm with the improved 2-OPT local search*

Рута 1
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 13 6 12 9 8 5
Укупно време трајања руте = 53.467
Укупна количина сакупљеног отпада у возилу = 59.40

Рута 2
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 4 3 15 17
Укупно време трајања руте = 55.899
Укупна количина сакупљеног отпада у возилу = 52.80

Рута 3
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 16 11 10 7 14
Укупно време трајања руте = 56.900
Укупна количина сакупљеног отпада у возилу = 53.90

Рута 4
Рута се формира од чворова. Након последњег чвора враћа се на базни чвор 1.
1 19 2 21 20 18
Укупно време трајања руте = 56.940
Укупна количина сакупљеног отпада у возилу = 57.20

Fig 4 *Route generation on the basis of the limitations of vehicle capacity and route duration time by applying the SA algorithm*

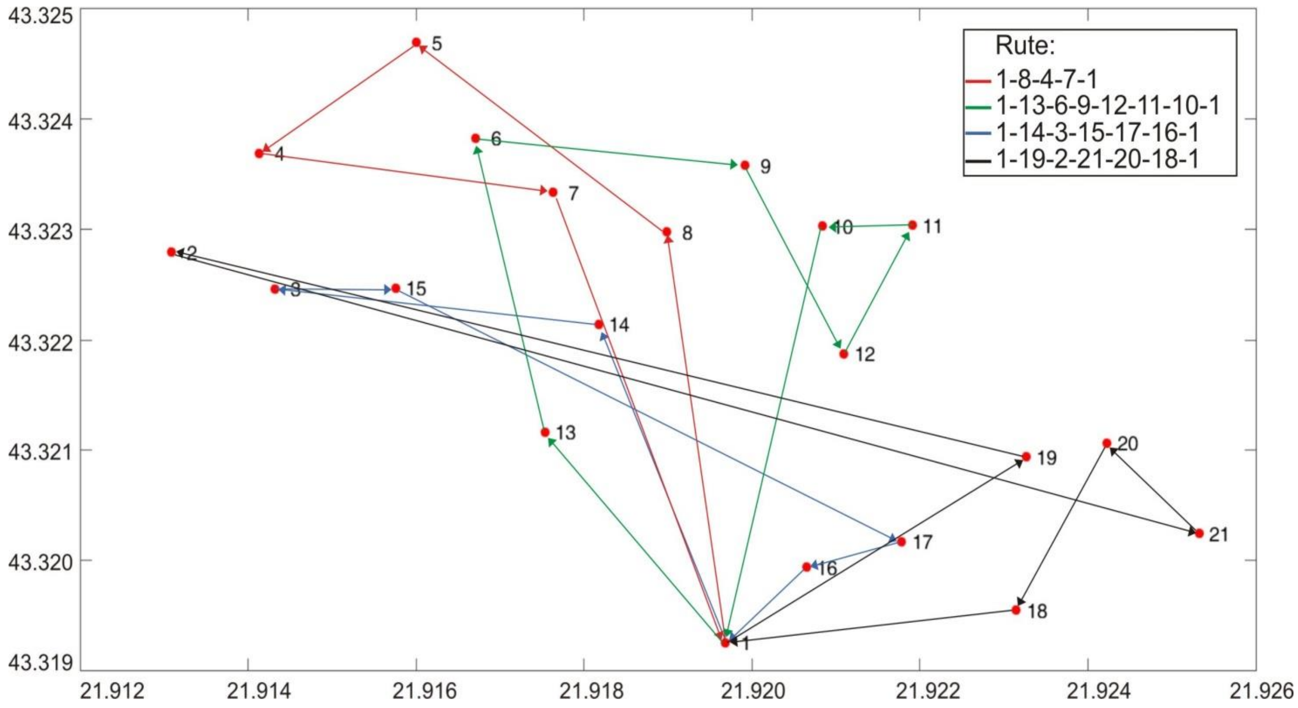

Fig. 5 *Diagram of routes obtained by the C-W algorithm with the improved 2-OPT local search*

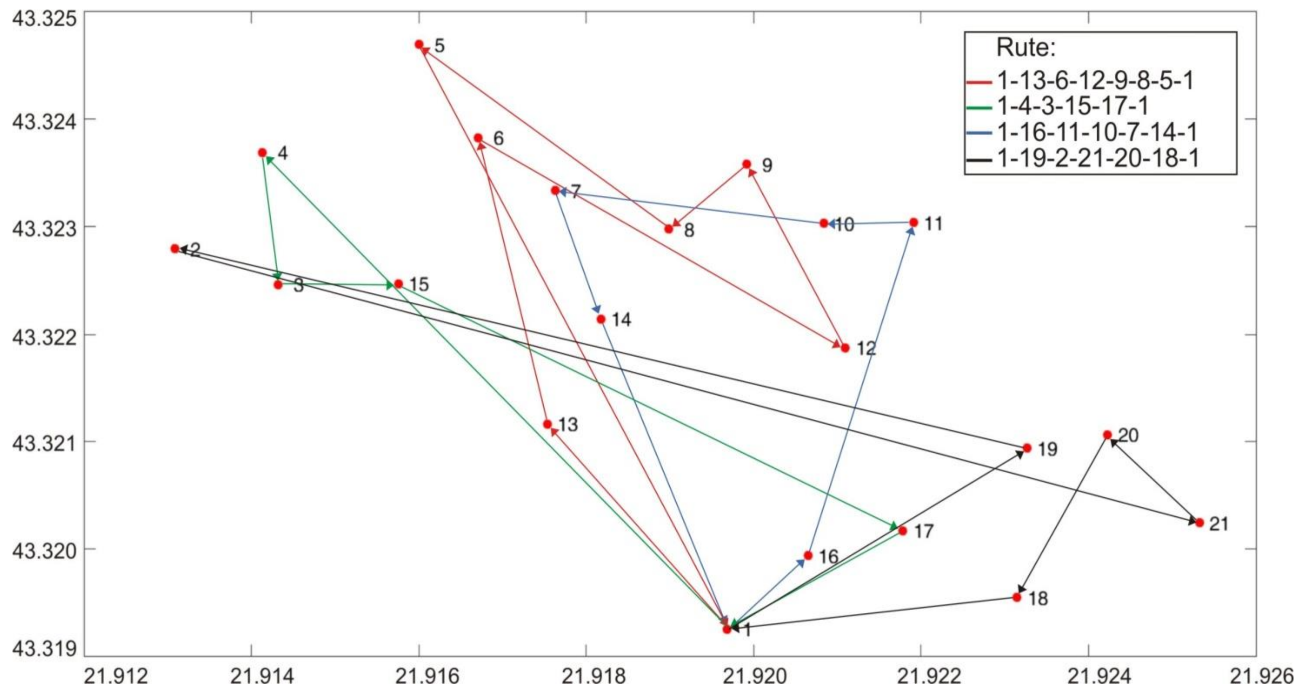

Fig. 6 *Diagram of routes obtained by the SA algorithm*

5 CONCLUSION

The problem of municipal waste collection was observed as the distance–constrained capacitated vehicle routing problem (DCCVRP). The planning and optimization of routes of municipal waste collection vehicles belongs to the operational level of an integrated waste management system. The optimization of waste collection and transport routes is, in fact, a permanent task of every public utility company, and this level of optimization should be controlled every five years as a rule. Correctly designed routes (optimal routes) represent an important economic and environmental segment for any company that deals with waste collection and transport.The paper showed that the application of heuristic and meta-heuristic methods can yield, in a reasonable time interval, sufficiently good i.e. optimal solutions. It can also be seen that the C-W algorithm provides decent initial solutions with a very small deviation after the improvement. Further research should be directed towards stochastic demands in nodes, i.e. it should include the assumption that the amount of municipal waste is not known in advance prior to the moment that a vehicle arrives at the location itself.

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