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SEPARATION OF CONVEX HULLS AND ITS APPLICATIONS IN THE MEDICINE AND ENGINEERING

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Abstract

In this paper, we study the problems which can appear in the work with large sets of data. Their analyzing is very important problem in modern science and practice. Here, we will consider the discrete problems, the sets of points provided by measuring and methods based on convex hulls. The absence of separability is one more difficulty in discussion. We will apply conclusions on methods for modeling expert medical prognostic systems and data mining.

Key words*: convex hull, separation hyperplane, predicting.*

1 INTRODUCTION

Multidimensional point sets are commonly used for modeling and analyzing data in computational geometry, mathematical optimization (see Boyd [1]) and theoretical computer science and many applied fields: robotics, graphics and vision, data mining, and machine learning. The ability to make difference between the nearest neighbors or convex hulls or partitioning of the space, is useful in algorithms for representing, manipulating, and filtering these kinds of data.

The computation of convex hull for a given set of points is hard problem in multidimensional spaces. Therefore, a large amount of research has gone into developing more efficient algorithms for solving the convex hull problem.

For every two disjoint convex hulls it exists separation strong, or sometimes weak hyperplane. Its finding is a problem in the multiple linear programming. But, when are given linearly inseparable sets, we consider several measures of how far they are from being separable.

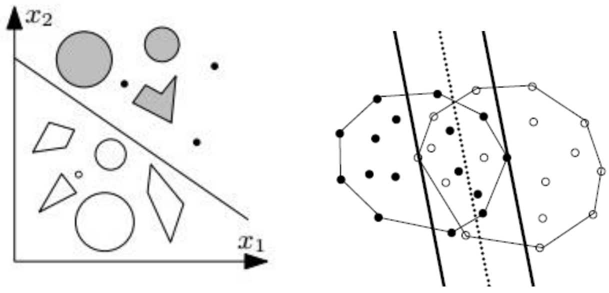


Fig. 1 *The separable and inseparable sets in* R^2

This methodology for constructing convex hulls and their separation can be used for modeling expert medical prognostic systems. Since we have linearly inseparable sets, we have to consider several measures of how far they are from being separable. In this paper, we will use data sets collected in the Gastroenterology and Hepatology Clinic in Niš.

2 THE HYPERPLANES

In the Euclidean space R^n, a set of n points $A=\{a_k=(a_{k1},a_{k2},\dots,a_{kn}):\ \ k=1,2,\dots,n\}$, determine the hyperplane

$$h[x,A]=\begin{vmatrix} x_1-a_{11} & x_1-a_{12} & \cdots & x_1-a_{1n} \\ a_{21}-a_{11} & a_{22}-a_{12} & & a_{2n}-a_{1n} \\ \vdots & & & \\ a_{n1}-a_{11} & a_{n2}-a_{12} & & a_{nn}-a_{1n} \end{vmatrix}=0, \qquad (1)$$

Which divide the space R^n plane into two half-spaces:

$$H^+[A]=\{x:h[x,A]>0\},\quad H^-[A]=\{x:h[x,A]<0\}. \quad (2)$$

For a set $U\supseteq A$ the hyperplane $h[x,A]$ the *separate hyperplane* if all its points are on one side of $h[x,A]$.

Let us denote by

$$\text{Sign}\, h[U,A]=\{\text{Sign}\, h[x,A]:\quad x\in U\}. \qquad (3)$$

This set can contain only the numbers $\{-1,0,1\}$. If

$$(\min \text{Sign}\, h[U,A])\cdot(\max \text{Sign}\, h[U,A])\neq -1, \qquad (4)$$

then $h[x,A]$ is a *separate hyperplane* for a set U.

Example 1. The set A={{1,4},{3,2}} determines the line $h[x_1,x_2,\{\{1,4\},\{3,2\}\}]\equiv x_1+x_2-5=0$ which is a hyperplane in R^2 and divides it into two halfspaces.

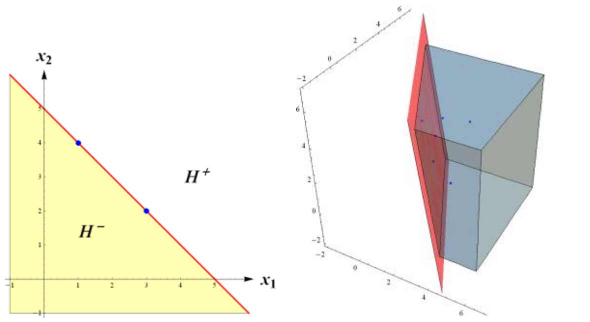


Fig. 2 *The hyperplanes and halfspaces in* R^2 *and* R^3.

CONVEX HULL

The *convex hull* of a finite set *U* is the smallest convex set which contains *U.* It is the cut of all halfspaces containing its points whuch are determined by the separate hyperplanes:

$$\mathrm{conv}(U) = \bigcap_{A \subseteq U \subseteq H^p[A]} H^p[A]. \tag{5}$$

Equivalently,

$$\mathrm{conv}(U) = \left\{ x = \sum_i \lambda_i u_i : \sum_i \lambda_i = 1 \wedge 0 \le \lambda_i \wedge u_i \in U, \forall i \right\}. \tag{6}$$

Example 2. The convex hull determined by the set

$$\mathrm{B} = \{\{1,4\},\{2,1\},\{2,2\},\{2,3\},\{3,2\},\{4,2\}\} \tag{7}$$

Here, a few interior points are interior points of convex hull.

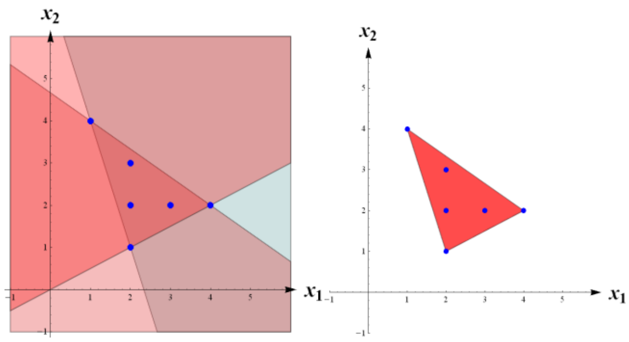


Fig. 3 *A convex hull*

3 THE SEPARATION OF CONVEX SETS

Let C and D be two sets in R^n. The *distance* between the sets C and D is

$$\mathrm{dist}(C,D) = \inf \{ \|u - v\| : u \in C, v \in D \}. \tag{8}$$

Theorem 3.1. *Let C and D be two closed convex sets in* R^n *with at least one of them bounded, and assume they are disjoined sets* $C \cap D = \varnothing$. *Then,* $a \in R^n : a \ne 0$ *and* $b \in R$ *exist such that*

$$a^T x > b, \forall x \in D, \quad a^T x < b, \forall x \in C. \tag{9}$$

Let $c \in C$ and $d \in D$ be points in which the infimum is achieved. Denote by

$$a = d - c, \quad b = \frac{\|d\|^2 - \|c\|^2}{2}. \tag{10}$$

Our separating hyperplane will be a function

$$f(x) = a \circ x - b = 0. \tag{11}$$

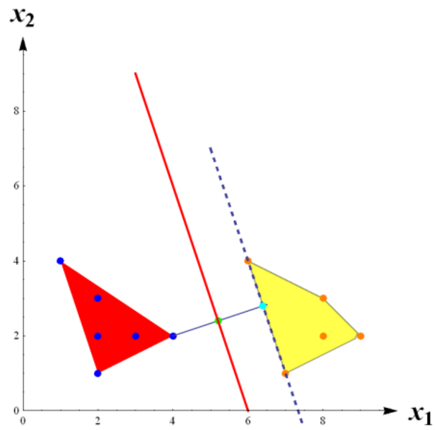


Fig. 4 *Separation of two convex hulls*

4 SEPARATORS OF INSEPARABLE SETS

Let *R* be a set of *r* red points and *B* a set of *b* blue points in R^n. Let *m* = *r* + *b* be the total number of points and assume that the point sets are disjoint and in general position, that is, no *n*+1 of the points lie in the same hyperplane in R^n. We say that *R* and *B* are linearly *separable* if there exists a linear separator, which is a hyperplane so that the red points lie to one its side and the blue points lie to the other. Let *h* be a hyperplane which divide the space into two halfspaces H^- and H^+. If *h* is a separator of *R* and *B*, we would have $R \subseteq H^-$ and $B \subseteq H^+$ or vice versa.
If there is no separator for *R* and *B*, then we say that the sets are *inseparable* (see Aronov [2] and Kalantari [3]). Equivalently, two sets *R* and *B* are inseparable if and only if the set

$$R - B = \mathrm{conv}\{u - v : u \in R, v \in B\} \tag{12}$$

contains the origin, i.e. $0 \in R - B$.
In that case, the sets misclassifies the red points $R(h) = R \setminus H^-$ and the blue points $B(h) = B \setminus H^+$. We use $T(h) = R(h) \cup B(h)$ to denote the set of points misclassified by *h*. We use *s(h)* to represent the quality of *h* as a classifier; it depends on *h* and $T(h)$. Our goal is to find a hyperplane that minimizes

$$s_2(h_{OPT}) = \min_h s_2(h), \tag{13}$$

under the following measure

$$s_2(h) = \sum_{p \in T(h)} d^2(p,h). \tag{14}$$

Here, $s_2(h)$ denotes the sum of squares of the Euclidean distances *d(p,h)* from *h* to points in $T(h)$.

4.1. One dimenisonal case

The input sets R and B lie on the real line. Then a classifier is a point h. We will assume that H^{+} is the half-line $[h,+\infty)$ and H^{-} is the half-line $(-\infty, h]$. In this case, the sum of squares of the distances from h to points in $T(h)$ is

$$s_2(h) = \sum_{p\in T(h)} (p-h)^2 = h^2|T(h)| - 2h\sum_{p\in T(h)} p + \sum_{p\in T(h)} p^2, \quad (15)$$

wherefrom

$$h_{OPT} = \frac{\sum_{p\in T(h)} p}{|T(h)|}. \quad (16)$$

Theorem 4.1. *The one-dimensional problem has a unique solution* h_{OPT} *which can be found in optimal linear time. It is the arithmetic mean (centroid) of the misclassified points.*

Example 2. The sets
r={{1,0},{3,0},{4,0},{6,0},{11,0}};
b={{5,0},{7,0},{8,0},{10,0},{12,0},{14,0},{15,0},{17,0}}
are inseparable. The dashed line in minimum of the sum of square distances of all points. The thick line is optimum of the sum of misclassified points.

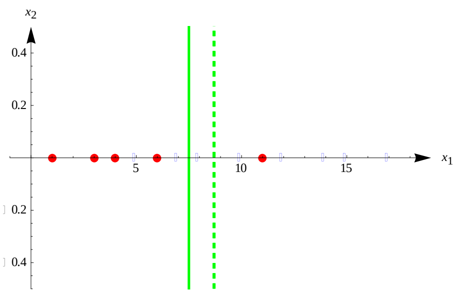


Fig. 5 *Separators of inseparable sets on R*

4.2. Multidimenisonal case

The previously mentioned method in multidimensional case requires a long lasting procedure.
The method of reducing convex hulls simply exclude the points of a convex set which belongs to another hull. We find the reduced convex hulls like it is shown on the Fig 4. If it is not enough, we have to exclude new points.

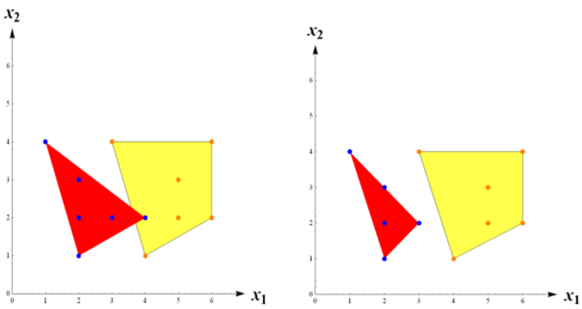


Fig. 6 *Reducing the convex hulls*

The another approach is based on the fact that it is undesirable to let one point to have excessive influence on the solution. Therefore, we want the solution to be based on a lot of points, not just a few bad ones. This can be done by *contracting* the convex hull by putting an upperbound on the multiplier in the convex combination for each point

$$\mathrm{conv}_K(U) = \left\{ x = \sum_i \lambda_i u_i : \sum_i \lambda_i \le K < 1 \wedge 0 \le \lambda_i \wedge u_i \in U, \forall i \right\}.$$

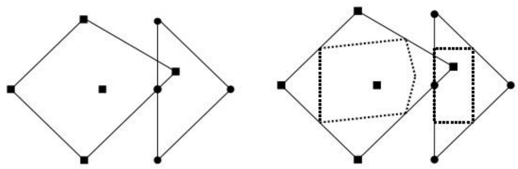


Fig. 7 *Conctracting the convex hulls*

5 A MEDICAL PROGNOSTIC SYSTEM WITH INSEPARABLE SETS OF DATA

This methodology for constructing convex hulls and their separation can be used for modeling medical prognostic systems. Acute pancreatitis is a serious disease of great importance in the clinical practice that may involve local tissues or affect other organs in a systemic manner, requiring, in such cases, an intensive care. The correct, early detected diagnosis and the determination of its severity are of fundamental importance for the appropriate therapeutic management of such patients. Laboratory and instrumental diagnostic markers and rating scales define problem where constructing convex hulls and their separation is much more complex problem (see, for example, Ivanchuk [4] and Simões [5]). Since we have linearly inseparable sets, we have to consider measures of how far they are from being separable. In this paper, we will use data sets collected in the Gastroenterology and Hepatology Clinic in Niš.

Table 1 Data for weak and severe pancreatitis.

TAFI		CRP	
Weak (blue)	Severe (red)	Weak (blue)	Severe (red)
1.10	1.23	2.9	2.3
1.83	3.03	3.5	5.3
2.06	3.50	8.9	11.3
2.13	4.53	13.8	16.0
2.33	4.70	18.4	26.3
2.40	4.97	20.5	37.2
5.10	5.30	21.1	39.7
5.90	6.23	36.7	136.3
6.13	7.30	41.9	115.4
7.20	9.57	110.1	206.5

D-dimer 2		Pt 1	
Weak (blue)	Severe (red)	Weak (blue)	Severe (red)
899	5212	96.0	37.9
336	260	63.0	100.0
689	536	78.2	97.3
522	4815	79.9	76.5
853	284	84.6	68.7
221	1687	78.2	82.7
2644	897	86.7	103.0
388	815	93.5	78.8
259	3416	47.4	81.8
4030	1501	60.3	67.2

INR 1	
Weak (blue)	Severe (red)
1.096	2.141
1.444	1.176
1.274	1.070
1.229	10.87
1.184	1.291
1.247	1.264
1.167	1.262
1.114	1.202
1.408	0.991
1.490	1.156

Fib 2	
Weak (blue)	Severe (red)
4.824	7.792
6.923	9.544
4.135	7.322
6.019	9.102
7.695	12.10
8.775	9.938
11.00	7.503
4.714	10.40
5.491	6.610
6.726	7.327

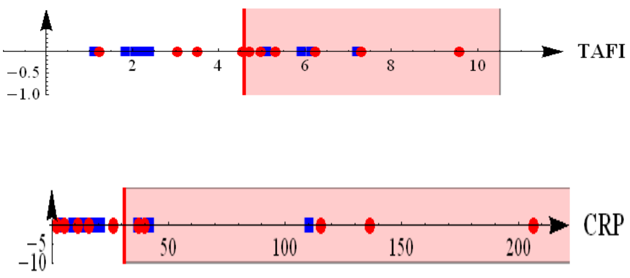
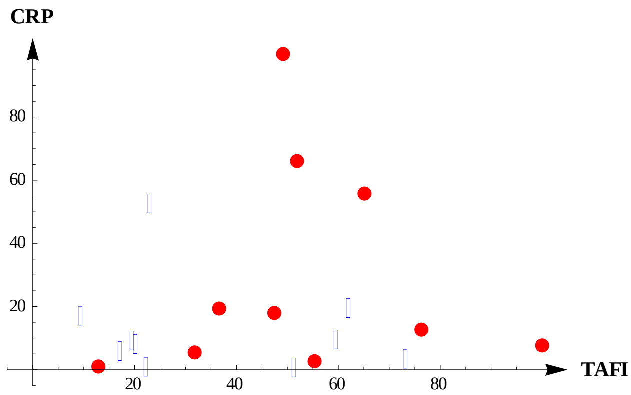


Fig. 9 *Searation of inseparable sets of data*

Table 1 Coclusions about pancratitis severity

Severe pancreatitis	
TAFI	> 4.571
CRP	> 30.89
D−dimer 2	> 1140
Pt1	> 77.61
APPT1	> 77.61
INR 1	> 1.2289
Fib 2	> 7.498

We have got positive recognition of pancratitis' severity is 70%. We believe that much better conclusions could be get if we consider this like a multidimensional problem.



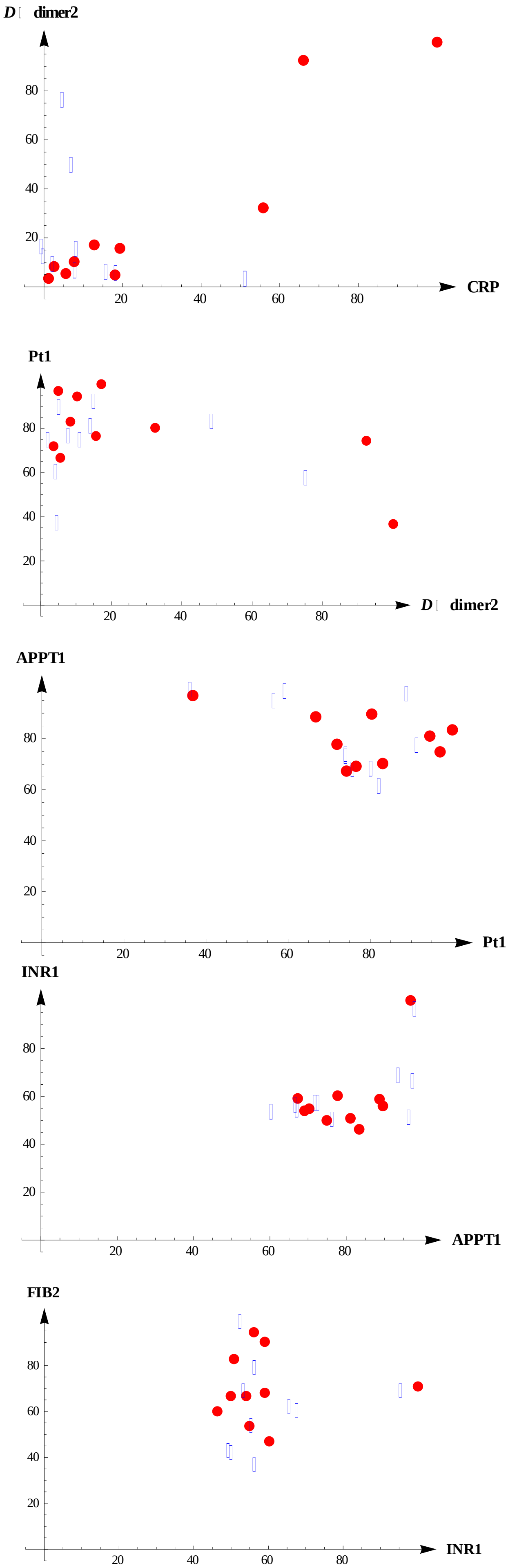


Fig. 9 *Comapison of two successive parameters*

6 OTHER APPLICATIONS

In this section, we will signify on applications of previous discussions in machine learning and data mining.

Example 6.1. Incremental learning with Revised Support Vector Regression with Filters (IRSVRF) consists of the following steps (see Son [6]):

1. Determine the optimal batch size based on generalization error rate and computing time for training.
2. Determine the reducing rates.
3. Train data in the first batch by the revised SVR.
4. Store data points representing support vectors in the next batch.
5. Train the data in the batch by the revised SVR, and obtain support vectors and corresponding regression function.
6. Store data points in the next batch, and go to step 5.

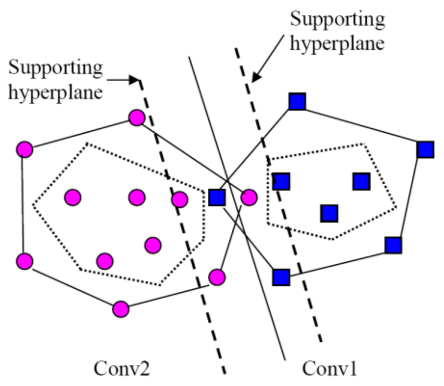
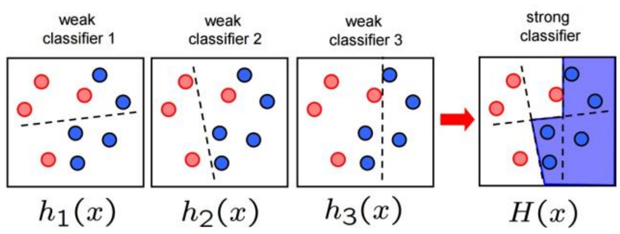


Fig. 8** Two filters for **IRSVRTF

Example 6.2. An algorithm for for constructing a strong classifier out of a linear combination of simple weak classifiers and setting the weights:

$$H(x) = \text{sign}\left(\lambda_1 h_1(x) + \lambda_2 h_2(x) + \lambda_3 h_3(x)\right), \quad h_k \in \{-1,1\}, \; k = 1,2,3. \tag{8}$$



Example 6.3. Fig. 10 illustrates how the parameters of square on the twodimensional dataset can be recognized. At every iteration the classifier becomes better than the previous one and finally converges to the correct square.

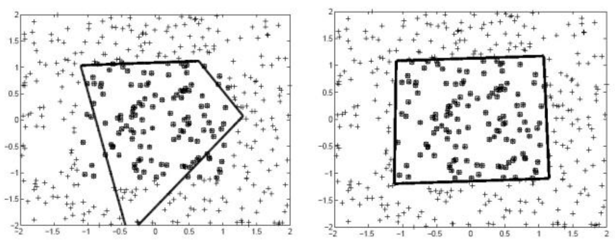


***Fig. 10** Learning square recognition*

7 CONCLUSION

This paper presents the methods for efficient work with large sets of data. Their analyzing is very important problem in modern science and practice. We consider the sets of points provided by measuring and methods based on convex hulls. The absence of separability is overcomed on a few ways. It is applyed in modeling expert medical prognostic systems, machine learning and data mining.

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