SEPARATION OF CONVEX HULLS AND ITS APPLICATIONS IN THE MEDICINE AND ENGINEERING

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Abstract

In this paper, we study the problems which can appear in the work with large sets of data. Their analyzing is very important problem in modern science and practice. Here, we will consider the discrete problems, the sets of points provided by measuring and methods based on convex hulls. The absence of separability is one more difficulty in discussion. We will apply conclusions on methods for constructing convex hulls and their problem in the multiple linear programming. But, when are given linearly inseparable sets, we consider several measures of how far they are from being separable. In this paper, we will use data sets collected in the Gastroenterology and Hepatology Clinic in Niš.

2 THE HYPERPLANES

In the Euclidean space $R^d$, a set of $n$ points $A = \{a_k = (a_{k1}, a_{k2}, ..., a_{kn}) : k = 1, 2, ..., n\}$, determine the hyperplane

$$h(x, A) = x_1 - a_{11} \quad x_2 - a_{21} \quad \cdots \quad x_n - a_{n1}$$

$$= a_{12} \quad a_{22} - a_{12} \quad a_{32} - a_{22} \quad \cdots \quad a_{n2} - a_{n1}$$

$$\vdots$$

$$a_{13} - a_{12} \quad a_{23} - a_{13} \quad a_{33} - a_{23} \quad \cdots \quad a_{n3} - a_{n2}$$

Which divide the space $R^d$ plane into two half-spaces:

$$H^+ [A] = \{x : h(x, A) > 0\}, \quad H^- [A] = \{x : h(x, A) < 0\}.$$ (2)

For a set $U \supset A$ the hyperplane $h(x, A)$ the separate hyperplane if all its points are on one side of $h(x, A)$.

Let us denote by

$$\text{Sign} \ h[U, A]\ = \text{Sign} \ h(x, A); \quad x \in \ U.$$ (3)

This set can contain only the numbers $\{-1, 0, 1\}$. If

$$\left(\min \text{Sign} \ h[U, A]\right) \cdot \left(\max \text{Sign} \ h[U, A]\right) \neq -1,$$ (4)

then $h(x, A)$ is a separate hyperplane for a set $U$.

Example 1. The set $A = \{1,4\}, \{3,2\}$ determines the line $h(x_1, x_2, \{1,4\}, \{3,2\}) = x_1 + x_2 - 5 = 0$ which is a hyperplane in $R^2$ and divides it into two halfspaces.
CONVEX HULL
The convex hull of a finite set \( U \) is the smallest convex set which contains \( U \). It is the cut of all halfspaces containing its points which are determined by the separate hyperplanes:

\[
\text{conv}(U) = \bigcap_{A \subseteq U, A \neq U} H^*[A].
\]

Equivalently,

\[
\text{conv}(U) = \left\{ x = \sum_i \lambda_i u_i : \sum_i \lambda_i = 1 \land 0 \leq \lambda_i \land u_i \in U, \forall i \right\}.
\]

Example 2. The convex hull determined by the set

\[
B = \{(1,4),(2,1),(2,2),(2,3),(3,2),(4,2)\}
\]

Here, a few interior points are interior points of convex hull.

3 THE SEPARATION OF CONVEX SETS
Let \( C \) and \( D \) be two sets in \( \mathbb{R}^n \). The distance between the sets \( C \) and \( D \) is

\[
\text{dist}(C, D) = \inf \{ \|u - v\| : u \in C, v \in D \}.
\]

Theorem 3.1. Let \( C \) and \( D \) be two closed convex sets in \( \mathbb{R}^n \) with at least one of them bounded, and assume they are disjoined sets \( C \cap D = \emptyset \). Then, \( a \in \mathbb{R}^n : a \neq 0 \) and \( b \in \mathbb{R} \) exist such that

\[
a^T x > b, \forall x \in D, \quad a^T x < b, \forall x \in C.
\]

4 SEPARATORS OF INSEPARABLE SETS
Let \( R \) be a set of \( r \) red points and \( B \) a set of \( b \) blue points in \( \mathbb{R}^n \). Let \( m = r + b \) be the total number of points and assume that the point sets are disjoint and in general position, that is, no \( n+1 \) of the points lie in the same hyperplane in \( \mathbb{R}^n \). We say that \( R \) and \( B \) are linearly separable if there exists a linear separator, which is a hyperplane so that the red points lie to one side and the blue points lie to the other. Let \( h \) be a hyperplane which divide the space into two halfspaces \( H^- \) and \( H^+ \). If \( h \) is a separator of \( R \) and \( B \), we would have \( R \subseteq H^- \) and \( B \subseteq H^+ \) or vice versa.

If there is no separator for \( R \) and \( B \), then we say that the sets are inseparable (see Aronov [2] and Kalantari [3]). Equivalently, two sets \( R \) and \( B \) are inseparable if and only if the set

\[
R - B = \text{conv}\{ u - v : u \in R, v \in B \}
\]

contains the origin, i.e. \( 0 \in R - B \).

In that case, the sets misclassify the red points \( R(h) = R \setminus H^- \) and the blue points \( B(h) = B \setminus H^+ \). We use \( T(h) = R(h) \setminus B(h) \) to denote the set of points misclassified by \( h \). We use \( s(h) \) to represent the quality of \( h \) as a classifier; it depends on \( h \) and \( T(h) \). Our goal is to find a hyperplane that minimizes

\[
s_i(h_{opt}) = \min_n s_i(h),
\]

under the following measure

\[
s_i(h) = \sum_{p \in T(h)} d^2(p, h).
\]

Here, \( s_i(h) \) denotes the sum of squares of the Euclidean distances \( d(p, h) \) from \( h \) to points in \( T(h) \).
4.1. One dimensional case

The input sets \( R \) and \( B \) lie on the real line. Then a classifier is a point \( h \). We will assume that \( H^+ \) is the half-line \([h, +\infty)\) and \( H^- \) is the half-line \((-\infty, h]\). In this case, the sum of squares of the distances from \( h \) to points in \( T(h) \) is

\[
s_i(h) = \sum_{p \in T(h)} (p - h)^2 = h^2 - 2h \sum_{p \in T(h)} p + \sum_{p \in T(h)} p^2,
\]

wherefrom

\[
h_{\text{opt}} = \frac{\sum_{p \in T(h)} p}{\|T(h)\|}.
\]

**Theorem 4.1.** The one-dimensional problem has a unique solution \( h_{\text{opt}} \) which can be found in optimal linear time. It is the arithmetic mean (centroid) of the misclassified points.

**Example 2.** The sets

- \( a = \{(1,0), (3,0), (4,0), (6,0), (11,0)\} \)
- \( b = \{(5,0), (7,0), (8,0), (10,0), (12,0), (14,0), (15,0), (17,0)\} \)

are inseparable. The dashed line in minimum of the sum of square distances of all points. The thick line is optimum of the sum of misclassified points.

4.2. Multidimensional case

The previously mentioned method in multidimensional case requires a long lasting procedure. The method of reducing convex hulls simply exclude the points of a convex set which belongs to another hull. We find the reduced convex hulls like it is shown on the Fig 4. If it is not enough, we have to exclude new points.

The other approach is based on the fact that it is undesirable to let one point to have excessive influence on the solution. Therefore, we want the solution to be based on a lot of points, not just a few bad ones. This can be done by contracting the convex hull by putting an upperbound on the multiplier in the convex combination for each point.

**Fig. 7 Constructing the convex hulls**

5 A MEDICAL PROGNOSTIC SYSTEM WITH INSEPARABLE SETS OF DATA

This methodology for constructing convex hulls and their separation can be used for modeling medical prognostic systems. Acute pancreatitis is a serious disease of great importance in the clinical practice that may involve local tissues or affect other organs in a systemic manner, requiring, in such cases, an intensive care. The correct, early detected diagnosis and the determination of its severity are of fundamental importance for the appropriate therapeutic management of such patients. Laboratory and instrumental diagnostic markers and rating scales define problem where constructing convex hulls and their separation is much more complex problem (see, for example, Ivanchuk [4] and Simões [5]). Since we have linearly inseparable sets, we have to consider measures of how far they are from being separable. In this paper, we will use data sets collected in the Gastroenterology and Hepatology Clinic in Niš.

**Table 1** Data for weak and severe pancreatitis.

<table>
<thead>
<tr>
<th>TAFI</th>
<th>Weak (blue)</th>
<th>Severe (red)</th>
<th>CRP</th>
<th>Weak (blue)</th>
<th>Severe (red)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak</td>
<td>1.23</td>
<td>1.10</td>
<td>Weak</td>
<td>2.3</td>
<td>2.9</td>
</tr>
<tr>
<td>Severe</td>
<td>3.03</td>
<td>2.13</td>
<td>Severe</td>
<td>16.0</td>
<td>13.8</td>
</tr>
<tr>
<td>Pt 1</td>
<td>103.0</td>
<td>15.4</td>
<td>Pt 1</td>
<td>206.5</td>
<td>110.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D-dimer 2</th>
<th>Weak (blue)</th>
<th>Severe (red)</th>
<th>Weak (blue)</th>
<th>Severe (red)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak</td>
<td>5212</td>
<td>899</td>
<td>Weak</td>
<td>37.9</td>
</tr>
<tr>
<td>Severe</td>
<td>512</td>
<td>336</td>
<td>Severe</td>
<td>100.0</td>
</tr>
<tr>
<td>Pt 1</td>
<td>1687</td>
<td>689</td>
<td>Pt 1</td>
<td>97.3</td>
</tr>
</tbody>
</table>

**Fig. 5 Separators of inseparable sets on \( R \)**

**Fig. 6 Reducing the convex hulls**
Table 1: Conclusions about pancreatitis severity

<table>
<thead>
<tr>
<th>Severe pancreatitis</th>
<th>TAFI</th>
<th>CRP</th>
<th>D-dimer 2</th>
<th>Pt1</th>
<th>APPT1</th>
<th>INR 1</th>
<th>Fib 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt; 4.571</td>
<td>&gt; 30.89</td>
<td>&gt; 1140</td>
<td>&gt; 77.61</td>
<td>&gt; 77.61</td>
<td>&gt; 1.2289</td>
<td>&gt; 7.498</td>
</tr>
</tbody>
</table>

We have got positive recognition of pancreatitis' severity is 70%. We believe that much better conclusions could be get if we consider this like a multidimensional problem.
6 OTHER APPLICATIONS

In this section, we will signify on applications of previous discussions in machine learning and data mining.

Example 6.1. Incremental learning with Revised Support Vector Regression with Filters (IRSVRF) consists of the following steps (see Son [6]):
1. Determine the optimal batch size based on generalization error rate and computing time for training.
2. Determine the reducing rates.
3. Train data in the first batch by the revised SVR.
4. Store data points representing support vectors in the next batch.
5. Train the data in the batch by the revised SVR, and obtain support vectors and corresponding regression function.
6. Store data points in the next batch, and go to step 5.

Example 6.2. An algorithm for constructing a strong classifier out of a linear combination of simple weak classifiers and setting the weights:

\[ H(x) = \text{sign} (\lambda_1 h_1(x) + \lambda_2 h_2(x) + \lambda_3 h_3(x)) \]

\[ h_i \in \{-1,1\}, \ k = 1,2,3 \] (8)

Example 6.3. Fig. 10 illustrates how the parameters of square on the twodimensional dataset can be recognized. At every iteration the classifier becomes better than the previous one and finally converges to the correct square.

7 CONCLUSION

This paper presents the methods for efficient work with large sets of data. Their analyzing is very important problem in modern science and practice. We consider the sets of points provided by measuring and methods based on convex hulls. The absence of separability is overcome on a few ways. It is applied in modeling expert medical prognostic systems, machine learning and data mining.

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REFERENCES


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