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SOLVING THE UNRELATED PARALLEL MACHINE SCHEDULING PROBLEM WITH SETUP TIMES

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Abstract

This paper presents the problem of unrelated parallel machines with setting time. Planning and scheduling jobs in parallel connection of machines belong to the group of difficult NP problems. Unrelated parallel machines with setting time appertain to the group of models within the planning and scheduling of jobs.. This application of the model in parallel connection of machines can often be seen in many practical examples. A genetic algorithm (GA) was used to solve the problem in the parallel connection of machines. The goal of this paper is to present a mathematical model to minimize the target function C_{max} *for the purpose of improving production productivity. The first part of the paper presents in detail the methodology which is used, while the second part presents the experimental results of research based on optimization. Based on the obtained results, the efficiency application of the genetic algorithm in solving planning problems in parallel connection of machines was confirmed.*

Key words*: parallel machine scheduling problem, genetic algorithm, setup times, optimization.*

1 INTRODUCTION

Resource planning and scheduling are important factors in a production system. In order for the production process itself to function properly, it is necessary to properly schedule jobs in the entire planning system. Today, the world of production cannot imagine one system without that system being arranged on the basis of various techniques in the field of artificial intelligence. Proper planning and scheduling of production resources include proper planning of production jobs and proper scheduling of all jobs at the right time in order to make the production process as efficient as possible. The problem of scheduling in production consists several phases and represents the correct schedule of several operations within several jobs on machines, all with the aim of minimizing the criterion function and higher productivity. Production planning is one of the basic phases of production system management where the basic planning goals, management strategies, and work methodologies are set in order for the system to be efficient and reach the initial criteria in the form of company goals. Planning is a very complex process and belongs to a group of complex and dynamic processes which are divided of several phases: long-term planning, medium-term planning, and short-term planning. All phases of planning depend on the planning process, goals, and strategy of a company [1], [2].

A key reason for planning and scheduling resources is to optimize overall planning time and maximize production productivity. The problem of resource planning and the methods used to solve this type of problem are gaining increasing application in many areas of logistics, planning, transportation and artificial intelligence.

The problem of planning and scheduling jobs has a great application in the production environment, so the proposed optimization of the problem, based on metaheuristic algorithm, we obtain graphical results in the form of a Gantt chart [3], [4], [5]. Based on a graphical presentation of the results obtained on the basis of optimization, a detailed schedule of all operations within the work as well as the total duration of all production processes during production.

In the second part of the paper, the methodology of the work as well as the mathematical model are presented in detail. The aim of this paper is to present the mathematical model parallel machine scheduling problem and the optimization approach in the form of metaheuristics for solving planning and scheduling problems in order to improve the quality of production.

Models in parallel connection of machines belong to the group of the most difficult optimization problems. The application of the model in parallel connection of machines in one production environment is presented in works such as [6], [7], [8], [9], [10].

2 METHODOLOGY

Metaheuristics is a higher level of heuristics in optimization whose goal is to select the optimal schedule jobs based on the proposed algorithm. Special cases and advantages of metaheuristic algorithms is to find the optimal solution in cases of imperfect information and in cases with special limitations where great effort is required during the optimization problem.

Compared to optimization algorithms and iterative methods, metaheuristics does not guarantee that a good enough optimal solution can be found for problems. In many cases, metaheuristics implement some kind of stochastic optimization, so the solution found depends on the set of random variables generated [11], [12].

In combinatorial optimization where used metaheuristic algorithms, optimal solutions with less computing power than optimization algorithms, iterative methods, or simple heuristics can usually be found. As such, they are useful in solving optimization problems. Figure 1 presents one of the ways of classifying metaheuristics [13], [14].

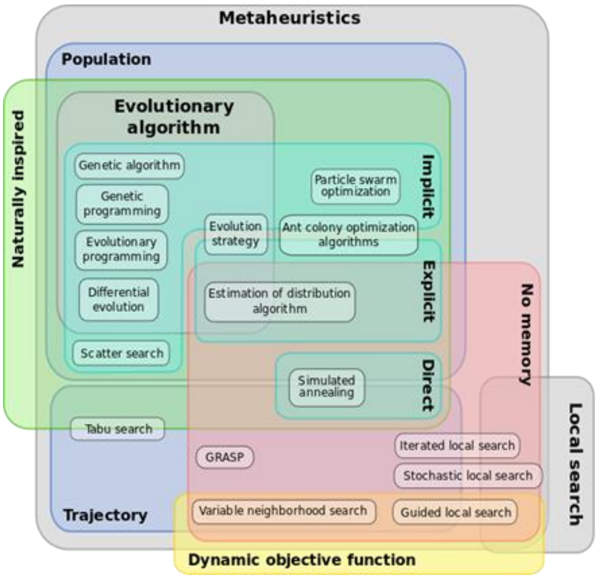

***Fig. 1** Different ways of classifying metaheuristics*

2.1 Genetic Algorithm

A genetic algorithm is an optimization technique used to solve nonlinear or non-differential optimization. The *GA* algorithm was developed by Holland in the 1970s. It is characterized by several phases in solving the defined problem in this case of planning and scheduling [9]. This algorithm mimics the natural process of selection, in the genetic structure changes, are possible by mutation of genetic material whose essence is the expansion of the search area and overcoming local extreme values. Crossing during the solution in *GA* is a process of combining several units to obtain a newly selected unit, this type is compared with a natural process such as parents and their offspring. New chromosomes inherit their parents' genes [11].

There are two types of crossing methods (operators) that are used for the exchange of genetic material, namely: Subsequence Exchange Crossover, which is the crossing of substrings on individual machines, and Job-Based Order Crossover crossing based on product order. When it comes to solving the problem of planning and scheduling, the most common examples of solving it are based on a genetic algorithm when we talk about the Parallel machine scheduling problem. One of the most common examples of solving is based on a coded job scheduling matrix and is used for scheduling problems on more than one machine [9].
Mutation involves a random change in the genes of individuals in a population. Mutations are generally applied at the level of genes of individuals in a population. This practically means a random change of route order. The main goal is to get an individual that cannot be obtained in other phases of the search. When the new main appearance of chromosomes is formed on the basis of the previous phase of crossing and mutation, the process of selecting their suitability follows. The processes of determining suitability, selection, crossing and mutation are performed until a predefined number of iterations is achieved. The basic goal is to get an individual that cannot be obtained in other phases.

The pseudo-code of the *GA* algorithm is presented in Table 1 [15].

Table 1 Pseudo code of Genetic Algorithm

Pseudo code of GA
Objective function $f_{(x)}$, $x = (x_1, \ldots, x_n)^T$
Encode the solution into chromosomes (binary strings)
Define fitness F (e.g., F ∞ f(x) for maximization)
Generate the initial population
Initial probabilities of crossover (p_c) and mutation (p_m)
While (t < Max number of generations)
Generate new solution by crossover and mutation
If p_c > rand, Crossover; **end if**
If p_m > rand, Mutate; **end if**
Accept the new solutions if their fitness increase
Select the current best for new generation
end while
Decode the results and visualization

3 MATHEMATICAL FORMULATION FOR THE PARALLEL MACHINE SCHEDULING PROBLEM

In the following chapter of this work, the model of job scheduling on a parallel set of machines will be presented. This type of deployment can be of great benefit in a production system during process optimization. In this case, the scheduling is done with a similar type of machine m where processing n jobs on a set of machines can be performed at the same time [16].
Formulation of the presented problem of parallel planning of machines with the times of performing can be defined in the following way. Let $N = \{1, \ldots, n\}$ be a set of jobs, and $M = \{1, \ldots, m\}$ a set of machines. This problem consists of scheduling *n* jobs on *m* machines with certain rules and restrictions during scheduling. An example of this type of job scheduling on a parallel set of machines can be seen in Figure 2 [17].

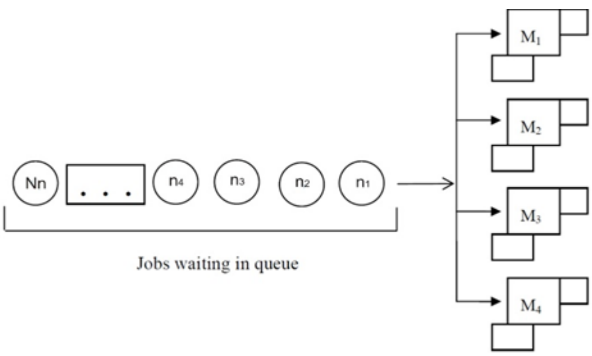

***Fig. 2** Graphic representation of the parallel machine scheduling problem*

The problem of planning jobs in parallel connection of machines consists in scheduling *n* jobs on *m* machines with certain rules and restrictions during scheduling. In the continuation of the paper, the used notation of the mathematical model is presented:

- *m* - machines,
- *n* - jobs,
- p_j - processing time,

Decision variables:

$$x_{ij} \begin{cases} 1 & if\ job\ j\ is\ processed\ on\ machine\ i \\ 0 & else \end{cases}$$

Function minimization:

$$min\,C_{max}$$

with constraints:

$$\sum_{j=1}^{n} x_{ij}p_j \le C_{max}, \qquad i = 1, \dots, m$$

$$p_j \le C_{max} \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1, \dots, n$$

$$x_{ij} \ge 0 \qquad i = 1, \dots, m; j = 1, \dots, n$$

a) the goal is to minimize the maximum end time of the schedule, the so-called makespan or also marked with C_{max},
b) each job $j \in N$ must be processed exactly once by only one machine $k \in M$,
c) each job $j \in N$ has a processing time p_{jk} which depends on the machine $k \in M$ where it will be assigned,
d) initiating the processing of a new job that may not have been available for execution until then *prmp*,
e) safe flow of machining operations on the entire machine line during machining *nwt*.

Based on the presented mathematical model with all limitations, this model can be represented as:

$$P_m \mid nwt, prmp \mid C_{max}$$

The described problem is interpreted as a problem of arranging *m* machines in a parallel connection, during the processing of jobs on machines. When processing work is interrupted, there is a possibility of processing a new available job at that moment, while the target function aims to minimize all jobs over time.

4 CASE STUDY

This chapter presents the problem of planning parallel machines in order to achieve maximum production productivity. Metaheuristics were used to optimize the jobs schedule, in order to achieve the best possible jobs schedule on the set of machines and to achieve maximum productivity. Within metaheuristics, the *GA* algorithm was used and based on the obtained results, the best work schedule based on the goal function was adopted. The mathematical formulation of the task is presented in detail in Section 3. Should be noted that the obtained solutions for each task are adopted based on the optimal result for the defined problem. Regarding the implementation of the *GA* algorithm, the optimization obtained experimental results based on testing in the programming language MATLAB R2015a, and numerical and graphical values were obtained on a computer HP ProBook 4530s with Intel (R) Celeron CPU B840 @ 1.90 GHz processor 1, 90 GHz and 6GM RAM.
Table 2 shows the processing times of all jobs for each machine separately. The setting times of these jobs on these machines are shown in Table 3.

Tabela 2 Processing times in machines M1 and M8

N	M1	M2	M3	M4	M5	M6	M7	M8
1	28	17	18	25	38	27	18	35
2	23	52	50	59	55	30	15	17
3	25	39	55	10	42	19	23	25
4	45	38	36	49	22	12	33	35
5	55	56	18	51	12	30	43	25
6	48	24	40	54	32	40	33	45
7	48	27	18	15	42	50	53	27
8	23	52	50	59	58	24	45	34
9	25	39	35	17	28	34	49	64
10	45	38	36	49	25	44	40	44

Tabela 3 Setup times in machine M1 and M8

M1	1	2	3	4	5	6	7	8	9	10	M2	1	2	3	4	5	6	7	8	9	10
1	5	7	5	7	7	5	2	4	3	8	**1**	7	7	7	6	3	2	4	7	2	5
2	3	5	8	5	6	6	5	2	7	4	**2**	7	7	3	4	4	3	3	6	6	3
3	6	8	6	8	3	2	7	8	4	2	**3**	7	2	2	8	2	5	3	7	2	2
4	3	4	3	6	6	6	8	8	5	5	**4**	2	5	7	3	6	3	2	6	7	5
5	2	7	3	6	2	4	3	8	3	4	**5**	6	4	6	6	3	7	2	5	8	3
6	7	4	4	7	6	2	3	8	3	3	**6**	5	5	7	6	2	8	6	6	7	8
7	3	7	7	8	3	5	5	7	6	3	**7**	3	4	6	4	7	2	8	5	2	2
8	5	7	7	8	7	3	6	5	4	8	**8**	2	4	4	2	4	5	2	4	2	4
9	6	4	7	2	8	2	4	3	8	6	**9**	7	3	4	2	6	2	4	7	6	5
10	4	3	4	8	8	3	3	4	2	5	**10**	3	3	7	4	4	7	8	8	7	7

M3	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**M4**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	6	5	8	5	4	6	3	8	2	3	**1**	7	7	8	3	8	2	5	2	3	8
2	4	6	5	6	5	5	5	6	3	2	**2**	8	5	2	3	6	7	6	4	7	6
3	5	8	5	7	4	3	2	5	2	6	**3**	3	3	2	4	4	4	8	8	4	6
4	6	7	4	5	7	6	7	7	5	8	**4**	7	5	4	8	7	7	3	5	4	7
5	8	4	5	3	7	2	7	5	3	8	**5**	3	5	6	5	8	5	7	4	3	2
6	7	7	5	5	5	6	8	5	4	3	**6**	8	8	5	4	5	5	6	3	7	8
7	2	2	2	4	6	6	8	6	4	5	**7**	7	7	8	3	5	2	5	5	6	4
8	6	2	8	8	8	2	5	4	2	4	**8**	4	8	5	8	4	2	8	8	7	2
9	5	2	2	6	7	2	3	3	5	5	**9**	7	6	5	4	2	2	4	2	7	7
10	5	3	5	6	6	3	2	6	6	3	**10**	5	4	5	7	3	6	3	5	2	7

Table 2 and Table 3 present the input parameters for the set mathematical model, also based on the table it can be concluded that it is a mathematical model with eight machines and ten jobs. In the further part of the paper, the experimental results are presented on the basis of research optimization and input parameters from Table 2 and Table 3. Graphical results in the form of jobs schedules on machines are graphically presented on the Figure 3 and 4.

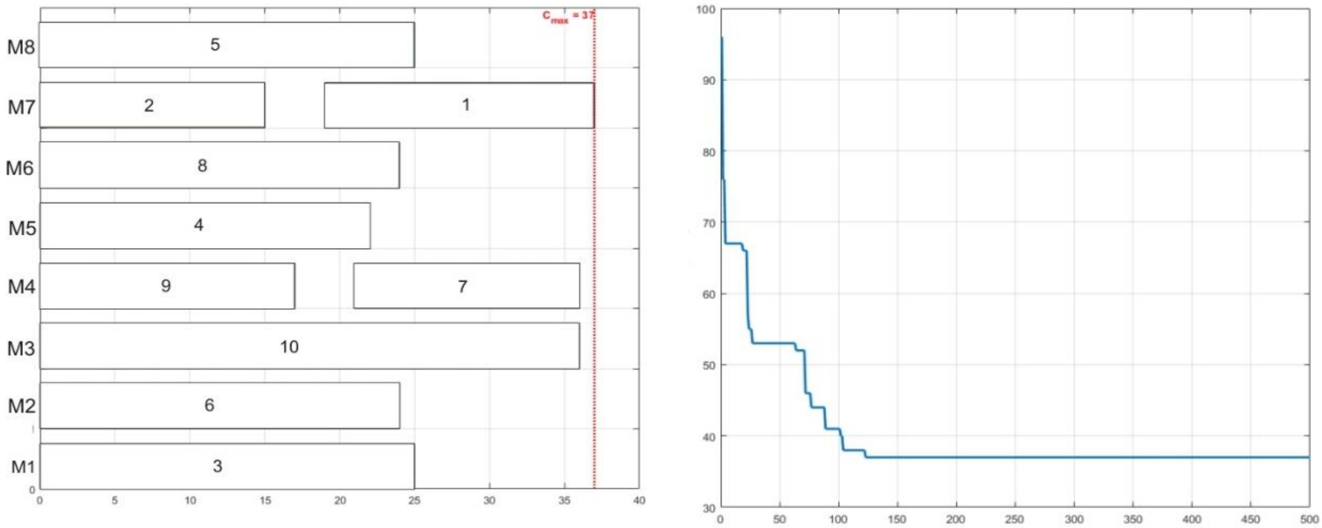

Fig. 3 *Graphical results based on GA algorithm: problem 10 x 8*

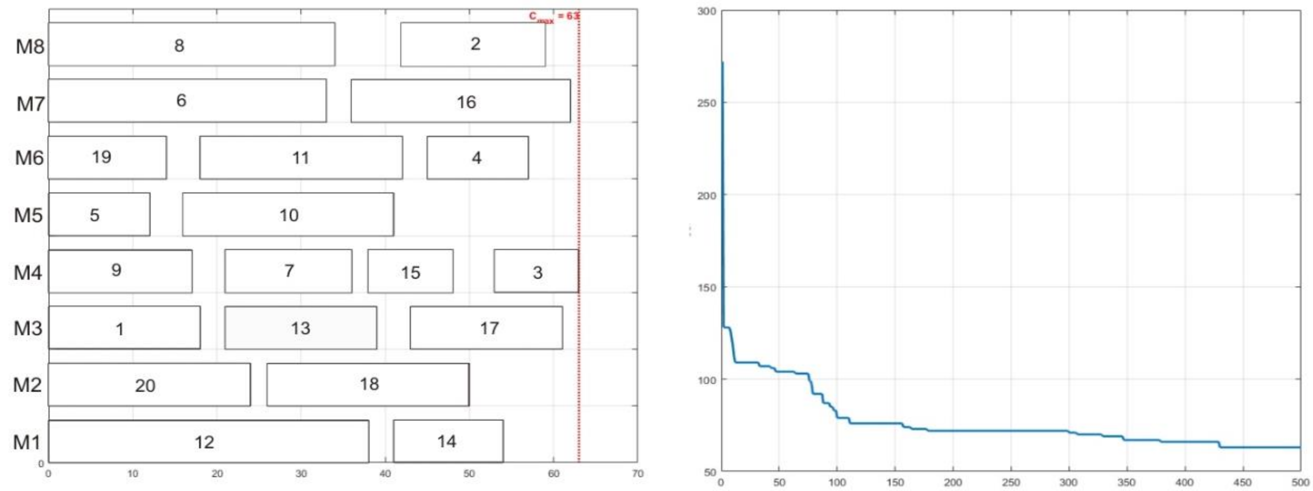

Fig. 4 *Graphical results based on GA algorithm: problem 20 x 8*

5 CONCLUSION

The problem of planning and scheduling resources is one of the most difficult problems in combinatorial optimization.

The process of planning and scheduling jobs in the production system is one of the most important factors in production. Increasing production productivity and optimizing sequential scheduling of jobs on machines is of great importance in the field of planning. The goal of this paper is to minimize the target function C_{max} for the purpose of improving production productivity.

Based on the input parameters presented in Tables 2 and 3, it can be concluded that the presented problem includes two sets of input data. For solving the unrelated parallel machine scheduling problem with setup times was used *GA*. The results of optimization based on the model presented in Chapter 3 are obtained in the form of a Gantt chart, in Figures 3 and 4. Based on the results shown in the Gantt chart, the efficiency of using the methodology is concluded. Testing of the proposed algorithm was performed based on optimization for various problems. The first data set consists of eight machines and ten jobs, while the second data set consists of eight machines and twenty jobs. Due to the range of input parameters, only the input parameters for a few machines are displayed. By analyzing the obtained results and comparing them with other algorithms in the literature, the conclusion is that the *GA* algorithm gives an optimal sequential schedule. For future case studies, the plan is to expand the mathematical model, to introduce additional constraints so that the process of planning and scheduling jobs is applicable in real-time.

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