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FORMING OF ZIPLINE MATHEMATICAL MODEL

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Abstract

Within this paper, the development of a mathematical model of a zipline system is presented, with particular emphasis on the influence of passenger swaying on travel ranges and velocities. Since passengers are typically attached to the trolley via harnesses, lateral and longitudinal swaying may occur during motion, potentially affecting the achieved ranges or velocities. The paper further provides results of conducted simulation, illustrating the extent to which passenger oscillations can influence overall motion characteristics.

Keywords: *zipline, mathematical model, sway*

1 INTRODUCTION

Considering that the issue of static analysis of the so-called "horizontal rope" is known from earlier and has been discussed in detail in [1], this chapter will provide only the most basic formulas necessary for this paper.

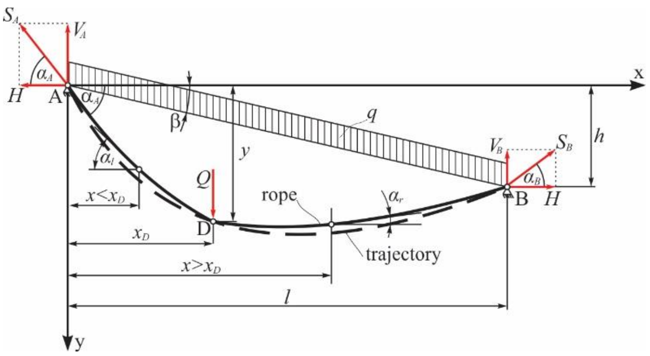

Fig. 1 *Parameters of rope loaded with own weight and concentrated load*

Fig. 1 shows a rope loaded with its own weight and with a concentrated load. If it is a rope that is fixed at one and tightened at the other end by weight, then the vertical coordinate of the load position, i.e. the concentrated mass (y) can be represented depending on the horizontal coordinate (x) as:

$$y = x \cdot tg\beta + \frac{x \cdot (l - x)}{2 \cdot H} \cdot \left(\frac{q}{\cos\beta} + \frac{2Q}{l} \right) \tag{1}$$

where:
β - inclination angle between stations coupling and horizontal line (-),
q - own weight of rope (N/m),
l - field span (m),
Q - rider weight (N),
H - horizontal component of the rope force (N).

In the case of a rope fixed at both ends, the vertical coordinate of the load position can be represented using the correction coefficient (k) as [2]:

$$y = x \cdot tg\beta + \frac{x \cdot (l - x)}{2 \cdot H} \cdot \left(\frac{q}{\cos\beta} + \frac{2Q}{l} \right) \cdot k \tag{2}$$

where:

$$k = \frac{1}{\sqrt{1 - \left(1 - k_1^2\right) \cdot k_2^2}} \tag{3}$$

$$k_1 = \frac{\cos\alpha_A}{\cos\beta} \cdot \frac{T}{H} \tag{4}$$

$$\text{tg}\alpha_A = \text{tg}\beta + \frac{q \cdot l}{2 \cdot T \cdot \cos\beta} \tag{5}$$

$$k_2 = \frac{2x}{l} - 1 \tag{6}$$

In the equations above T represents the force of the unloaded rope (the case when the transverse force Q isn't acting) which is obtained using an iterative procedure based on the following nonlinear equation [3]:

$$\frac{q^2 \cdot l^3}{24 \cdot T^2} - \frac{T \cdot l}{E \cdot A_u} - \frac{\left(Q + \frac{q \cdot l}{2} \right)^2 \cdot l}{8 \cdot H^2} - \frac{q^2 \cdot l^3}{96 \cdot H^2} + \frac{H \cdot l}{E \cdot A_u} = 0 \tag{7}$$

where:
E - elastic modulus of rope (MPa),
A_u - cross-sectional area of rope (mm^2).

2 MODEL FORMATION

In order to determine the dynamics of the ziplines rider's movement (law of motion), the equilibrium of all forces in the current position of the trolley will be observed. When forming a mathematical model, it must be borne in mind that in the case of a load sliding down a rope, the second-order theory must be used due to the large displacements which occur during movement [4, 5].

The relevant mathematical model, which is shown in Fig. 2, will be formed while neglecting small quantities of higher order.
The so-called static trajectory of the load is determined by expressions (1) and (2), where according to [6-8] for "flat" catenary the influence of the rope oscillation in the vertical plane is relatively small and can be ignored.

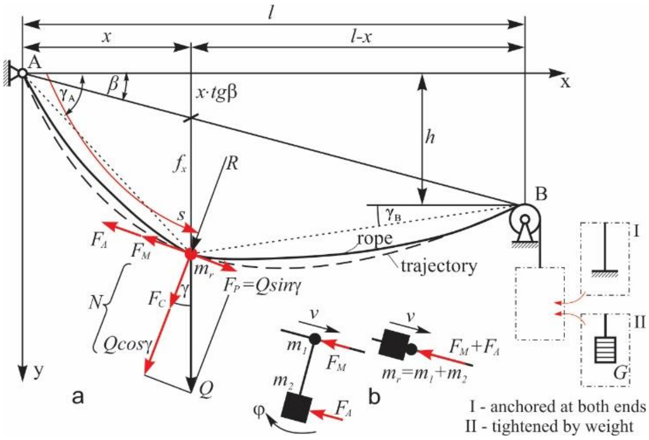

***Fig. 2** Mathematical model: a) movement of concentrated mass, b) mass reduction*

A rider connected to a trolley by belts forms a mathematical pendulum, but if the launch is correct (jerkless start), and if the length of the belts is short, the swing effect can also be ignored [9]. Accordingly, the mathematical model, Fig. 2b, can be represented as movement of a concentrated mass along a trajectory determined for static conditions. During movement, the mechanical and air resistances are acting onto the concentrated mass. They are always acting in the direction that is opposite to the direction of movement. The wind is also acting on the mass with variable speed and direction but its influence can be viewed integrally as aerodynamical resistances.
The movement of the concentrated mass (m) is opposed by the mechanical and air resistances at all times, while the movement is caused by the component of weight force. The resistance forces are acting in the direction of the tangent to the trajectory at the point where the material point is currently located. Centrifugal force and the component of weight force, which are making normal reaction (N), perpendicular to the tangent, are leading to mechanical resistance.
The current value of the inclination angle of the tangent to the trajectory (γ) can be determined based on Fig. 2a, for the case where the rope is tightened by weight, so that the "pulling force", i.e. the component of the concentrated mass which leads to movement amounts to:

$$F_P = Q \cdot \sin\gamma \tag{8}$$

It should be noted that γ represents the angle of inclination of the tangent to the trajectory, while α represents the angle of inclination of the tangent to the rope itself. The curve of trajectory, unlike the rope itself, is not discontinuous at the position of the trolley. The angle γ, can be obtained even as the derivative of the position function:

$$tg\gamma = \frac{dy}{dx} = tg\beta + \frac{(l-2x)}{2\cdot H}\cdot\left(\frac{q}{\cos\beta} + \frac{2Q}{l}\right) \tag{9}$$

or based on the known values of the angles of the left (11) and right (12) sides of the rope:

$$\gamma = \frac{\alpha_l + \alpha_r}{2} \tag{10}$$

where:

$$tg\alpha_l = tg\beta + \frac{l-2x}{2H}\cdot\left(\frac{q}{\cos\beta} + \frac{2Q}{l}\cdot\frac{l-x_D}{l-x}\right) + \frac{x}{H}\cdot\frac{Q}{l}\cdot\frac{l-x_D}{l-x} \tag{11}$$

$$tg\alpha_r = tg\beta + \frac{l-2x}{2H}\cdot\left(\frac{q}{\cos\beta} + 2\cdot\frac{Q}{l}\cdot\frac{x_D}{x}\right) - \frac{l-x}{H}\cdot\frac{Q}{l}\cdot\frac{x_D}{x} \tag{12}$$

Since the movement is performed along a curved path, a centrifugal force occurs with an intensity of:

$$F_C = \frac{m}{R}\cdot v^2 \tag{13}$$

where R represents radius of curved path:

$$R = \frac{H}{q + \frac{2Q}{l}} \tag{14}$$

The mechanical resistance then amounts to:

$$F_M = f\cdot(Q\cdot\cos\gamma + F_C) \tag{15}$$

and the aerodynamic resistance:

$$F_A = \frac{1}{2}\cdot\rho\cdot c_d\cdot A\cdot(v \pm v_w)^2 \tag{16}$$

The equation of motion of a body mass m follows from Newton's second law:

$$m\cdot\vec{a} = \left(\vec{F}_P - sign(v)\cdot\left(\vec{F}_M + \vec{F}_A\right)\right) \tag{17}$$

whence the acceleration of the body could be expressed as:

$$a = \frac{d^2s}{dt^2} = \frac{1}{m}\cdot\left\{Q\cdot\sin\gamma - sign(v)\cdot\left[f\cdot\left(Q\cdot\cos\gamma + \frac{m}{R}\cdot v^2\right) + \frac{1}{2}\cdot\rho\cdot c_d\cdot A\cdot(v\pm v_w)^2\right]\right\} \tag{18}$$

where s represents the arc length.
In the case of a rope anchored at both ends, the angle of inclination of the tangent cannot be expressed explicitly but is obtained by differentiating expression (2) under the condition that Eq. (7) is satisfied, so the obtained value is included in expression (18) and finally can be integrated to obtain velocity and travelled path.
In the case of the rope tightened by weight, the tangent inclination angle can be expressed explicitly, so by including the expression for the tangent inclination angle (9), Eq. (18) would take the form:

$$
\begin{aligned}
a = \frac{d^2 s}{dt^2} = \frac{1}{m} \cdot \Bigg\{ & Q \cdot \sin \operatorname{arctg}\left(\operatorname{tg}\beta + \frac{(l-2x)}{2 \cdot H} \cdot \left(\frac{q}{\cos\beta} + \frac{2Q}{l} \right) \right) - \\
& - sign\left(\frac{ds}{dt}\right) \cdot \left[f \cdot \left(Q \cdot \cos \operatorname{arctg}\left(\operatorname{tg}\beta + \frac{(l-2x)}{2 \cdot H} \cdot \left(\frac{q}{\cos\beta} + \frac{2Q}{l} \right) \right) + \frac{m}{R} \cdot \left(\frac{ds}{dt}\right)^2 \right) + \right. \\
& \left. + \frac{1}{2} \cdot \rho \cdot c_d \cdot A \cdot \left(\frac{ds}{dt} \pm v_w \right)^2 \right] \Bigg\}
\end{aligned}
\qquad (19)
$$

Velocity and travelled path are obtained by integrating expression (19). Bearing in mind that s can be represented as a function of x, it's visible that in the mentioned equation the second derivative of the variable depends on the variable itself, its first derivative, the sign of the first derivative and the square of the first derivative, whereby the variable itself also appears as an argument of trigonometric and inverse trigonometric functions. In other words, it is a nonlinear differential equation of the second order which can be solved numerically.

3 SWING EFFECT

One of the assumptions used in the formation of the mathematical model is that the impact of passenger sway on the ranges is small, and that it can be neglected. This was partially supported by [10], however, within this point it will also be supported by simulations performed with a specialized software for dynamic simulations *MSC ADAMS*. Namely, the software will create simulations of passenger descent in which passengers will be able to sway in relation to the trolley (hereinafter *first type of simulation*), and simulations in which the mass of the passenger will be reduced to the trolley, i.e. the trolley and passenger will be viewed as a concentrated mass (hereinafter *second type of simulation*). By comparing the results of the above simulations, it can be seen how much influence the swaying of the load has.

The simulations were performed for the zipline described in [1] and the case of two passengers with a total mass of 190 kg and a trolley with a mass of 10 kg, with the assumed frontal areas of the passengers of 0.6 m^2 and the trolley of 0.1 m^2. The trolley and the passengers are simplified and represented as spheres. The length of the belts with which the passengers are attached to the trolley is 1.5 m. The trolley is set to move along a predetermined trajectory. The trajectory is drawn according to the expression (1) using the *MS Excel* software, and then *imported* into the *MSC ADAMS* software. Since the trajectory depends only on the total mass, and not on the passenger-trolley distribution, the same trajectory can be used for both cases. The connection between the sphere representing the trolley and the trajectory is of the *Point-Curve-Constraint* type and at any time it is possible to read the value of the normal reaction, which is then used to calculate the mechanical resistance to the movement. In the first type of simulation, the sphere representing the passenger is connected to the sphere representing the trolley by a massles rod that has pivot connections with the spheres at both ends. A schematic representation of the model is given in Fig 3.

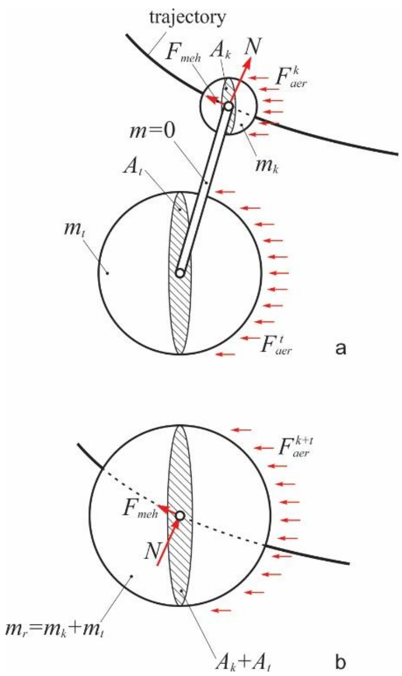

Fig. 3 *Schematic representation of the model for: a) first type of simulation, b) second type of simulation*

In addition to mechanical resistance to motion, the body is also affected by aerodynamic resistance, which is calculated based on the current speed of the body. In the first type of simulation, the sphere representing the trolley is acted upon by a force F^k_{aer}, and the sphere representing the passengers is acted upon by a force F^t_{aer}, while in the second type of simulation, the sphere of mass m_r is acted upon by a force F^{k+t}_{aer}. The appearance of the model in the *MSC ADAMS* software is given in Fig. 4.

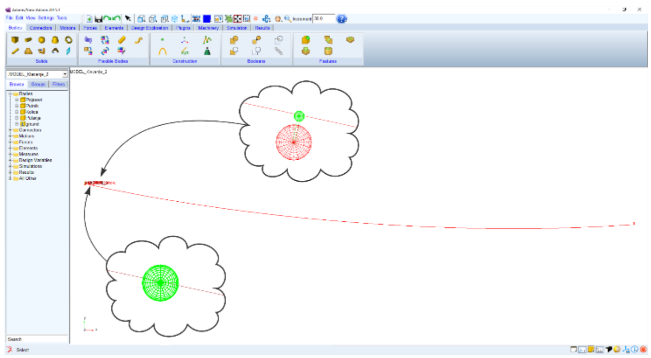

Fig. 4 *Model within the MSC ADAMS software*

The simulation results are shown in the Figs. 5 and 6. From the Fig. 5 it can be seen that the velocity of the trolley oscillates in relation to the velocity of the passenger, however, there is no significant deviation in the velocity of the passenger in the case when the entire mass is concentrated at the location of the trolley compared to the case when the mass of the passenger can oscillate in relation to the trolley.

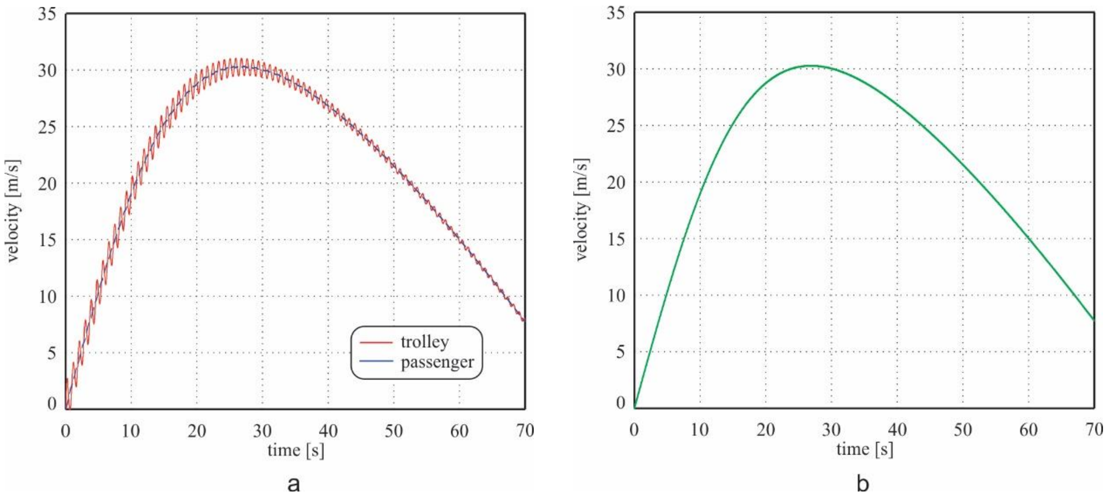

Fig. 5 *Achieved velocities according to simulation for:*
a) first type, b) second type

Fig. 6 shows the ranges in the case where the passenger is allowed to swing relative to the trolley, and in the case where the masses of the passenger and trolley are concentrated at the location of the trolley.

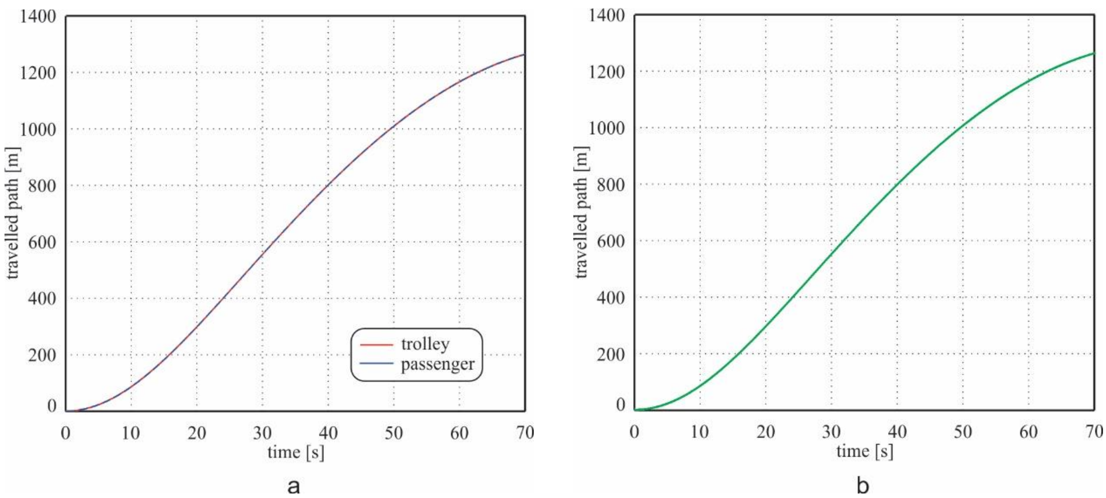

Fig. 6 *Travelled path according to simulation for:*
a) first type, b) second type

The diagrams shown justify neglecting the influence of the swaying of passengers relative to the trolley during movement for real values of the masses of passengers and trolleys, as well as real values of the connecting belts length.

4 CONCLUSION

This paper presents the development of a mathematical model of a zipline, designed to provide a fast and straightforward way to determine the passenger's kinematic characteristics. In addition to describing the model itself and outlining its formulation based on known equations and parameters, the paper also examines the influence of passenger swing during motion. Specifically, simulations of passenger motion, with and without swinging, were performed using the specialized dynamic simulation software MSC ADAMS. The simulation results show that, for realistic values of the parameters of the given zipline, the effect of swinging can be considered negligible. However, it is important to note that this simplification is valid only for ziplines with a relatively small inclination angle and should not be applied to all configurations.

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