

SELECTION OF THE OPTIMAL THREE STAGE PLANETARY GEAR TRAIN FOR APPLICATION AT ROAD VEHICLE WINCH

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Abstract

This study investigates the application of multicriteria optimization to the design of a planetary gear train intended for use in an automotive winch system. A three-stage planetary gear train, composed of fundamental planetary gear configurations, is examined under conditions representative of real transmission operation. The optimization framework is based on a proprietary algorithm integrated into the PlanGears software. The algorithm utilizes approximated analytical models for key performance indicators, including volume, mass, efficiency, and manufacturing cost. The design variables considered in the optimization process are the numbers of teeth, the number of satellites, the module, and the face width. Geometric and strength-related requirements necessary for reliable system performance are expressed through a set of functional constraints.

Key words: optimal solution, planetary gear train, road vehicle winch

1 INTRODUCTION

In recent years, planetary gear trains (PGTs) have gained increasing attention across a wide range of industrial applications. Their growing popularity is primarily attributed to their inherent advantages, including compact construction, high load-carrying capacity, and the favorable distribution of transmitted power among multiple planet gears. These characteristics enable the development of gear units that combine high power density with extensive flexibility in achieving various transmission ratios. However, the vast diversity of possible kinematic configurations, together with the relatively complex

analytical procedures required compared to conventional gear mechanisms, highlights the need for systematic research aimed at fully exploiting the performance potential of planetary gear systems.

This paper presents the application of multicriteria optimization to the design of a planetary gear train intended for use in a road-vehicle winch. Considering the specific operating conditions of the transmission system, a three-stage planetary gear train composed of basic planetary gear configurations is analyzed. The optimization procedure relies on a dedicated algorithm implemented within the PlanGears software, which incorporates approximate analytical expressions for key performance metrics and design constraints.

2 ROAD VEHICLE WINCH STRUCTURE

All-terrain vehicles are intended for off-road traffic and often end up in a trap from which they cannot escape under their own power. The winch, as an additional equipment of the off-road vehicle, is used to pull the vehicle out of the trap with the help of an external force. This winch, like a winch in the general sense, consists of a drive motor, a drum, and a mechanical transmission between them.

The development of a mechanical transmission involves establishing the characteristics of the working and driving machine and providing an insight into what kind of transformation is needed between them. In this case, the driving machine is the electric motor, and the working machine is the winch drum.

A winch has two fundamental performance characteristics: pulling force and pulling speed. The pulling force represents the maximum vehicle mass the winch can draw under specified operating conditions. In essence, it corresponds to the maximum load that can be applied to the rope and the entire winch structure while still ensuring proper functionality as designed, expressed in terms of the mass being pulled. For easier selection by end users, manufacturers typically specify this value as the mass the winch can pull, rather than the force itself. Therefore, this parameter can also be referred to as the pulling capacity.

To determine the operating conditions of the transmission, data from real working and on-road machines used in off-road vehicles were used [1].

The Warn 9.5XP winch is a product of the American company Warn, designed for installation on off-road vehicles. It is typically mounted at the front of the vehicle, although rear installation is also possible depending on the application. The winch consists of an electric motor, a brake, a power transmission system, a drum, a steel cable (wire rope), a fairlead, and the electrical components required for connection and control, as is shown in Fig. 1.



Fig. 1 Warn 9.5 XP winch

The intention in this paper is to determine the design parameters of the planetary gear train stages, by using the software PlanGears, according to multicriteria optimization. The input data is defined according the data from [1]. In this way it is possible to check the program accuracy and possibility.

3 STRUCTURES OF BASIC TYPE OF PGT

Three stage transmission shown in the Fig. 1 consists of basic type of planetary gear train which is given in the Fig. 2.

The basic type of planetary gear train (PGT), i.e. a design which has a central sun gear (external gear - 1), ring gear (internal gear - 3), satellites (planets - 2) and carrier (h), shown in Fig. 2, is the subject of the paper, limited to geared pairs. Planets are in simultaneous contact with the sun gear and the ring gear.

This type of PGT is often used as single stage transmission, as a building block for higher compound planetary gear trains.

Its advantage over other PGT types lies in its efficiency. The efficiency value varies negligibly in the whole range of the internal gear ratio ($p = |z_3|/z_1$). Also, this type has small overall dimensions and mass, and its production costs are relatively low because of the relatively simplified production.

It is appropriate to show a basic planetary gear train with the Wolf-Arnaudov's symbol, Fig. 2c [2÷4].

By using this symbol, the train shafts are shown with different width lines and a circle. Sun gear shaft 1 is shown by a thin line, ring gear shaft 3 by a thick line and the carrier shaft h by two parallel lines since the carrier shaft is summary element regarding by carrier stopping negative transmission ratio is obtained.

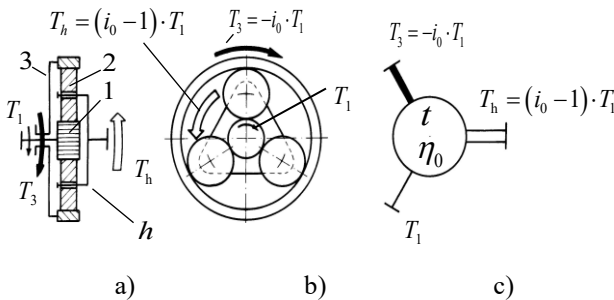


Fig. 2 Wolf-Arnaudov's symbol and torque ratios of the basic type of PGT

Planetary gear train shafts are loaded with torques whose ratio is also shown in Fig. 2. Torques on the ring gear shaft T_3 and on the carrier shaft T_h are given as a function of the ideal torque ratio t and the torque on the sun gear shaft T_1 [2,3].

Ideal torque ratio:

$$t = \frac{T_3}{T_1} = \frac{T_{D\max}}{T_{D\min}} = \frac{|z_3|}{z_1} > +1 \quad (1)$$

Torques:

$$T_1 : T_3 : T_h = +1 : +t : -(1+t) \quad (2)$$

It is assumed:

$$\eta_0 = \eta_{13(h)} = \eta_{31(h)} = 1 \quad (3)$$

4 MATHEMATICAL MODEL FOR PGT OPTIMIZATION

The process of finding the optimal solution starts by defining a mathematical model [6÷7].

An optimization task is defined by the variables, objective functions and conditions required for the proper functioning of a system determined by the functional constraints.

The complete procedure of optimal solution selection is described in [6÷7], and now will be briefly pointed. Under the mathematical model definition, it is necessary to determine the objective functions, functional constraints and variables since each objective function is the function of several parameters. The following characteristics are chosen for objective functions of a planetary gear train: volume, mass, efficiency and production cost of gear pairs.

$$V = \frac{\pi}{4} \cdot b \cdot \left(\frac{m_n \cdot z_3}{\cos \beta} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt23}} \right)^2 \quad (4)$$

$$m = 0.25 \cdot \pi \cdot b \cdot \rho \cdot \frac{m_n^2}{\cos^2 \beta} \cdot \left[k_1 \cdot z_1^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt12}} + n_w \cdot k_2 \cdot z_2^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt12}} + k_3 \cdot z_3^2 \cdot \frac{\cos^2 \alpha_t}{\cos^2 \alpha_{wt23}} \right] \quad (5)$$

$$\eta = \frac{1 - i_0 \cdot \eta_0}{1 - \eta_0} \quad (5)$$

$$F_T = T_{P1} + n_w \cdot T_{P2} + T_{P3} \quad (6)$$

Then, mathematical model of nonlinear multi-criteria problem in concrete task, can be formulated as follows:

$$\begin{aligned} & \max \{f_1(x), f_2(x), f_3(x), f_4(x)\} \\ & \text{subject to } x \in S \end{aligned} \quad (7)$$

where:

$$\begin{aligned} f_1(x) &= -V(x), f_2(x) = -m(x), \\ f_3(x) &= \eta(x), f_4(x) = -T(x) \end{aligned}$$

The most important criterion for selecting these "equally good" solutions is the *Pareto optimality concept*: Solution $x \in S$ is Pareto optimal if there is no solution $y \in S$ such that holds $f_i(x) \leq f_i(y)$ for all $i=1, \dots, n$ and for at least one index i holds strict inequality, i.e. $f_i(x) < f_i(y)$. Determination of the Pareto optimal solutions set is the first step in optimal solution finding. Next step is optimal solution choice from Pareto set. In this model, weighted coefficients method is applied for choosing optimal solution from Pareto solutions [6,7].

Thus, some additional information is needed to be able to select one of the solutions as the final solution. This final decision is made by a corresponding *scalarized problem*.

In this method the following scalarized problem is constructed:

$$\begin{aligned} & \max f^M(x) = w_1 f_1^0(x) + \dots + w_m f_m^0(x) \\ & \text{s.t. } x \in S \end{aligned} \quad (8)$$

Here, the weighted coefficients (or weights) w_i are positive real numbers and $f_i^0(x) = (f_i^0)^{-1} f_i(x)$ are normalized objective functions where f_i^0 are normalizing coefficients. In this approach, the components of the ideal point $f^* = (f_1^*, f_2^*, f_3^*, f_4^*)$ are used as normalizing coefficients, i.e. $f_i^0 = f_i^*$ for $i=1, 2, 3, 4$. Therefore,

absolute values of all objective functions are between 0 and 1, which simplifies the choice of the weighted coefficients. All solutions obtained by this method are Pareto optimal. In this paper, the following variables are considered: the number of teeth of the central sun gear z_1 , the number of teeth of planets z_2 , the number of teeth of the ring gear z_3 , the number of satellites n_w , the gear module m_n and the facewidth b . Functional constraints are related to mounting conditions, geometrical conditions and strength conditions.

Optimization procedure

For the given input data (input number of revolution, input torque, service life in hours, application factor, accuracy grade (Q-DIN3961), minimal safety factor - flank, minimal safety factor - root, gear materials, allowed deviation of gear ratio, range of z_1 variation), all 6-tuples of design parameters $(z_1, z_2, z_3, n_w, m_n, b)$ satisfying the functional constraints are generated and the values of the objective functions for every 6-tuple are computed. A set of feasible solutions is obtained. Then, the set of Pareto solutions is created.

Next, it is necessary to choose only one optimal solution among all the generated solutions. The weighted coefficient method for solving multicriteria optimization problem is predicted.

5 NUMERICAL EXAMPLE

The development of a mechanical transmission involves establishing the characteristics of the working and driving machine and providing an insight into what kind of transformation is needed between them. In this case, the driving machine is the electric motor, and the working machine is the winch drum.

The input data is according to [1].

This winch uses a direct current electric motor with a series excitation power of 4.5 kW [52] as a power unit. The total power used during operation is 1.5 kW at 1500 min⁻¹. It is powered by electricity from the vehicle's battery (accumulator) with a voltage of 12 V. The engine draws a current of 482 A at maximum load. The electric motor is connected by a hexagonal shaft to the planetary gear transmission. The shaft goes from one end where the motor is located through the drum where it has the brake when the motor is not active, then through the entire transmission all the way to the input gear at the other end. The brake is located inside the drum and when the engine is not running it brakes the drum in relation to the input shaft of the transmission. The drum is connected by direct toothing at the output of the transmission, so that when the brake is active, it brakes the output shaft in relation to the input shaft of the transmission. Thus, the transmission takes all the load with the winch loaded. The brake is released when the electric motor is activated, and the load is then transferred to it.

The transmission output speed is obtained based on the expected speed of the rope and the diameter of the drum. From the traction characteristics of this winch, the maximum traction force (mass) is 4310 kg at a pulling speed of 1.89 m/min. Considering the output speed

determined in this way and the input speed from the drive motor, the required transmission ratio is obtained: $i = 156$.

Due to their characteristics, planetary gear trains are suitable for that usage. First, mass and volume are considerable advantages, e.g., when exploring the different possibilities for placing the transmission on the vehicle.

Taking into consideration working conditions and the purpose of the machine, the authors of this paper opted for the distribution of transmission ratios in reducers used in the turning system of cranes and similar devices [8].

$$i_1 \approx 1.12 \cdot \sqrt[3]{i_{\text{tot}}} = 1.12 \cdot \sqrt[3]{156} = 6.03$$

$$i_2 \approx \sqrt[3]{i_{\text{tot}}} = \sqrt[3]{156} = 5.38$$

$$i_3 \approx 0.89 \cdot \sqrt[3]{i_{\text{tot}}} = 0.89 \cdot \sqrt[3]{156} = 4.79$$

The first stage

The important required input data are as follows: $i = 6.03$, $n_{\text{in}} = 1500 \text{ min}^{-1}$, $P = 4.5 \text{ kW}$, $T_{\text{in}} = 28.6 \text{ Nm}$, $L = 2000 \text{ h}$, $K_A = 1.25$, IT7 for all gears, the sun and planet gear material is 16MnCr5, while 42CrMo4 is predicted for the ring gear, the range for the number of teeth of the sun gear is $12 \div 36$, and tolerances for the gear ratio is $\pm 3\%$.

The feasible set consists of 446 solutions. The number of Pareto solutions is 25. The multicriteria optimization is carried out using the weighted coefficient method

Since the main objective functions are minimization of overall transmission and mass, along with production costs reduction, and ensuring reliability and durability, the following weighting factors adopted: $w_1=0.4$, $w_2=0.4$, $w_3=0.1$, $w_4=0.1$.

The optimal solution obtained in this way is shown in Table 1 with a set of objective functions values shown in Table 2.

Table 1 Optimal solution for first stage obtained by weighted coefficient method

Variable values					
$x_1=z_1$	$x_2=z_2$	$x_3=z_3$	$x_4=n_w$	$x_5=m_n$	$x_6=b$
12	23	-60	3	2.25	14

Table 2 First stage objective functions

$f_1 \text{ (mm}^3\text{)}$	$f_2 \text{ (kg)}$	f_3	$f_4 \text{ (min)}$
194.421	0.936	0.97	66.93

The second stage

Since the output of the first stage is the input of the second stage, the next input data is chosen for the multi-criteria optimization application: $i=5.38$, $n_{\text{in}}=250 \text{ min}^{-1}$, $T_{\text{in}}=171.882 \text{ Nm}$, $L=2000 \text{ h}$, IT7 for all gears, sun gear material/satellite material/ring gear material are 20MnCr4/20MnCr4/34CrNiMo6, $\Delta i=3\%$, $z_1=12 \div 26$.

The feasible set consists of 1823 feasible solutions. The number of Pareto solutions is 67. By application weighted coefficient method with weighted coefficient: $w_1=0.4$, $w_2=0.4$, $w_3=0.1$, $w_4=0.1$ the solution shown in Table 3 is obtained, with a set of objective functions values shown in Table 4.

Table 3 Optimal solution for second stage obtained by weighted coefficient method

Values of the Variables					
z_1	z_2	z_3	n_w	m_n	b
12	19	-51	3	2.75	23

Table 4 Objective function for solution shown in table 3

f_1 (mm ³)	f_2 (kg)	f_3	f_4 (min)
355,323.112	1.7	0.978	75.024

The third stage

Since the output from the second stage directly drives the third stage, the input dataset for the multi-criteria optimization of this stage is determined as follows: $i = 4.95$, $n_{in} = 47.619 \text{ min}^{-1}$, $T_{in} = 902.38 \text{ Nm}$, $L = 2000 \text{ h}$, $K_A = 1.25$, IT7 for all gears, the sun and planet gear material is 16MnCr5, while 42CrMo4 is predicted for the ring gear, the range for the number of teeth of the sun gear is $12 \div 36$, and tolerances for the gear ratio is $\pm 3\%$. The optimization process generated 2203 feasible solutions, among which 67 are Pareto-optimal. Through the application of the weighted coefficients method, following the same procedure as in the first and second stages, the resulting solution is given in Table 5, with its objective function values provided in Table 6.

Table 5 Optimal solution for third stage obtained by weighted coefficient method

Values of the Variables					
z_1	z_2	z_3	n_w	m_n	b
13	17	-50	3	4.5	40

Table 6 Third stage objective functions

f_1 (mm ³)	f_2 (kg)	f_3	f_4 (min)
1,495,501.95	7.425	0.977	133.529

The real value of the whole transmission ratio is $i_r = 152.77$, which means tolerances for the gear ratio is $\pm 2.7\%$, what is within the permissible limits.

It can be concluded that the selected design parameters for all three stages of the planetary gear train satisfy the requirements defined in the input data.

This three-stage transmission variant is acceptable, as it achieves a sufficiently high overall efficiency, small overall dimensions and mass and provides objective function values that meet all additional design requirements.

6 CONCLUSION

In this paper, an original model for multicriteria optimization of a three-stage planetary gear train intended for use in a road-vehicle winch is presented. The main objective functions were to minimize overall dimensions and mass, increase torque capacity, and reduce production costs and ensuring reliability and durability.

The compound transmission system consists of three basic planetary gear stages. Optimal solutions are obtained for

each stage individually, forming a set of Pareto-optimal designs. The weighted-coefficient method is applied to select the final optimal configuration from the Pareto set. To determine the operating conditions of the transmission, data from real working and on-road machines used in off-road vehicles were used, which also makes physical sense of optimal solution.

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