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SENSITIVITY ANALYSIS OF FLEXURE HINGE DYNAMICS USING TAGUCHI DESIGN, ANOVA, AND MONTE CARLO SIMULATION

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Abstract

Unlike traditional joints, flexure hinges rely on elastic deformation, which provides motion of a so-called compliant mechanism. This elastic deformation makes their dynamic characteristics highly sensitive to geometric and material variations. This study integrates the Taguchi design of experiments with two statistical and probabilistic methodologies: analysis of variance (ANOVA) and Monte Carlo simulation. The idea is to compare these methods in order to quantify the influence of key geometric parameters on the first natural frequency of a flexure hinge with a circular notch. Results highlights the notch radius as the critical design parameter and underscores the value of probabilistic approaches in mechanical design optimization.

***Keywords**: compliant mechanisms, flexure hinges, natural frequency, taguchi design, anova, monte carlo simulation.*

1 INTRODUCTION

In the modern domains of mechanical engineering and micro-mechanical engineering, the demand for precisely engineered, compact and reliable compliant mechanisms is becoming more and more apparent. Unlike traditional joints, these innovative systems use elastic deformation of flexure hinges, which allows them to have a monolithic structure, eliminate gaps and minimize maintenance costs [1]. Nevertheless, the very dependence on elastic properties makes their dynamic attributes - predominantly natural frequencies - very susceptible to minor changes in geometric configurations and material properties. Lack of awareness of the spectrum of these variations can cause resonant phenomena, increased material fatigue and ultimately catastrophic failure of the mechanism.

In paper [2], an analytical framework is developed for calculating the natural frequencies of flexure hinges with variable cross-sections by discretizing the hinge into a series of constant-section beam segments. Several discretization strategies are evaluated, the optimal segment count is identified, and the influence of mass on frequency prediction is examined. The method is validated on a compliant four-bar linkage, demonstrating that the proposed approach provides sufficiently accurate results for compliant mechanisms with non-uniform flexure hinges.

Paper [3] introduces a new pseudo-rigid-body model with 3-DOF joints for analyzing the free vibrations of planar compliant mechanisms composed of rigid links and flexure hinges. By modeling each hinge with lateral and rotational springs, the method predicts natural frequencies while examining the influence of different hinge compliance formulations. A numerical example of an RRR compliant micro-motion stage demonstrates that the proposed model provides higher accuracy compared to the classical 1-DOF pseudo-rigid-body approach.

Study in paper [4] employs the Taguchi method and FEA to optimize the geometry of a flexure-hinge tensural displacement amplifier, focusing on how hinge thickness and hinge angle influence the first natural frequency. The analyses (S/N, ANOVA, and regression) consistently show that increasing hinge thickness significantly raises the natural frequency, while a larger hinge angle reduces it. The optimized frequency of 214.06 Hz closely matches the predicted value of 198.18 Hz, confirming both the validity and accuracy of the proposed optimization approach.

Paper [5] uses the Taguchi method and FEA to optimize the design parameters of a compliant mechanism with the goal of maximizing its first natural frequency. The results show that hinge thickness and width are the most influential factors, followed by hinge angle, input body length, and fillet radius, as confirmed by S/N and ANOVA analyses. The developed model achieves an R^2 of 99.84% and a maximum first-mode frequency of 821.83 Hz, demonstrating strong predictive accuracy and effective parameter optimization.

Paper [6] proposes a multi-response optimization framework for a two-degree-of-freedom compliant mechanism using a hybrid methodology that combines the Taguchi method, response surface modeling, grey relational analysis, and entropy weighting. By varying hinge thicknesses, the study simultaneously maximizes displacement and the first natural frequency, with ANOVA and confirmation tests validating the accuracy of the developed mathematical models. The optimized results show strong agreement between experimental data and ANSYS simulations, demonstrating the method's effectiveness for micro-positioning compliant mechanisms.

In this paper, probability and statistics methods were used to quantitatively assess the influence of three key parameters - joint thickness (h), segment length (b) and circular arc radius (R) - on the first natural frequency of the mechanism. The aim of this paper is to illustrate how the integration of Taguchi experimental design, ANOVA statistical analysis and Monte Carlo methodology facilitates a thorough and computationally efficient investigation of the robustness of compliant mechanisms.

2 FLEXURE HINGES

Every mechanism consists of multiple members connected by joints. A member of a mechanism is a group of elements that are interconnected and function as a whole, meaning they are considered a single rigid body. A joint connection enables motion between contacting members, thereby providing the mechanism's functionality. The number of possible relative movements in conventional joints (revolute, double-revolute, prismatic, revolute–prismatic, rolling, spherical, and others) is defined by the number of degrees of freedom of the joint.

A flexure hinge can be described as an elastic segment of small thickness placed between two rigid segments. This structure enables relative motion between those segments, and mechanisms composed of such hinges are called compliant mechanisms. Flexure hinges represent an innovation in mechanism design, as they can significantly reduce some of the disadvantages of traditional joints [1].

The classification of flexure hinges within flexure mechanisms can be performed according to the nature of deformation into two basic types: concentrated compliance and distributed compliance. In the case of concentrated compliance, deformation occurs mainly in a localized, narrower zone of the hinge, while in distributed compliance, deformation is more evenly spread along a larger portion of the mechanism's constituent elements.

Basic classification of flexure hinges according to their shape:

- Notch-type flexure hinges,
- Leaf-spring flexure hinges,
- Beam-type flexure hinges.

Notch-type flexure hinges represent a special category of hinges where the concentration of deformation is determined by the notch geometry. These hinges can be further classified according to the shape of the notch (Fig. 1):

- Flexure hinges with a circular notch (Fig. 1a),
- Flexure hinges with a rectangular notch (Fig. 1b and c),
- Flexure hinges with an elliptical notch (Fig. 1d),
- Flexure hinges with a hyperbolic notch (Fig. 1e),
- Flexure hinges with a parabolic notch (Fig. f),
- Flexure hinges with a notch whose shape is defined by polynomial functions (Fig. 1g and h).

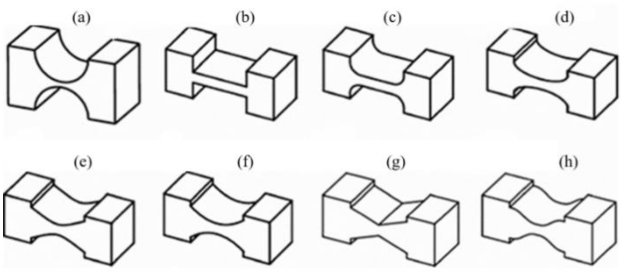

Fig. 1 *Type of notch-type flexure hinges*

3 FLEXURE HINGES GEOMETRY

The flexure hinge selected for investigation in this study is a circular-notch flexure hinge, modeled in SpaceClaim, which is part of the Ansys 19.0 software package. The geometry is parameterized by three dimensions:

- Parameter 1: h (hinge thickness),
- Parameter 2: b (hinge height),
- Parameter 3: R (hinge radius).

The objective of this investigation is to determine which parameter has the greatest influence on the natural frequency of this mechanism. Natural frequencies refer to the characteristic frequencies at which a system tends to oscillate in the absence of external forces or damping effects.

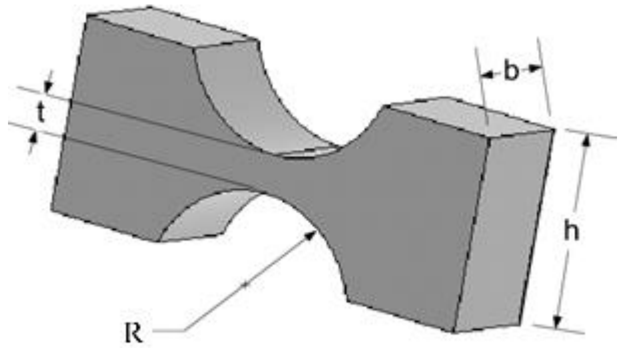

Fig. 2 *Dimensions of circular-notch flexure hinge*

The nominal values were chosen as $h = 10.5$ *mm*, $b = 5$ *mm*, and $R = 5$ *mm*. For the design of experiments (Taguchi L9), three levels were used for each parameter within a technologically feasible range (Table 1). These levels were selected to ensure that the geometry remains physically meaningful. The condition $h > 2R$ ensures that notch overlap does not occur, which would compromise both physical realizability and the validity of the investigation. Care was taken during the selection of parameter values to avoid violation of this condition.

Table 1 Nominal values for each parameter

Parameter	*Nominal values*		
h [mm]	10.5	11	11.5
b [mm]	5	4.5	5.5
R [mm]	5	4	4.5

The model was parameterized in Ansys SpaceClaim using driving dimensions and subsequently imported into Ansys Workbench, where the Modal analysis simulation was performed. The material used was Aluminum Alloy with the following properties: Young's modulus of 71,000 MPa and Poisson's ratio of 0.33. A single boundary condition was applied, consisting of a Fixed Support on one side of the hinge. The mesh was generated using Solid186 elements, with local mesh refinement in the notch region due to the expected higher stress concentrations. In the Solution section, Total Deformation was defined as the output parameter,

Table 2 *Results of ANOVA analisys*

Source	*DF*	*Seq SS*	*Contribution [%]*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
C1	1	7944	0.22%	7944	7944	9.61	0.032
C2	1	659	0.02%	659	659	0.71	0.437
C3	1	3556245	99.63%	3556245	3556245	3854.38	0
Error	5	4613	0.13%	4613	923		

providing the natural frequencies and the corresponding mode shapes (Fig. 3).

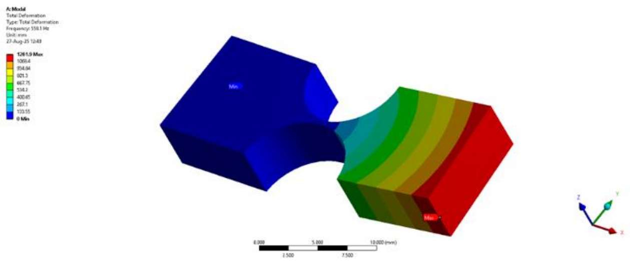

Fig. 3 *Total deformation*

The objective of the design of experiments is to reliably assess the influence of the principal geometric parameters on the first natural frequency while minimizing the number of required simulations [7-10]. A Taguchi orthogonal array L9 was employed, as shown in Fig. 4. All parameter combinations were selected such that the geometric condition $h > 2R$ is satisfied.

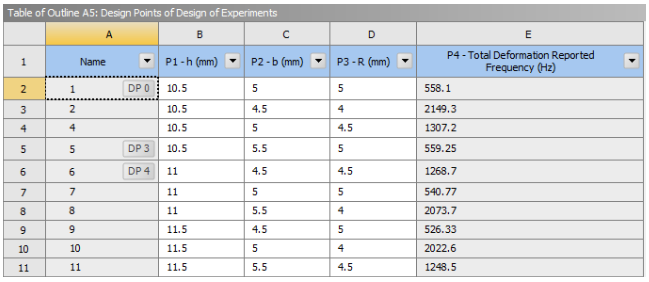

Table of Outline A5: Design Points of Design of Experiments

	A	B	C	D	E
1	Name	P1 - h (mm)	P2 - b (mm)	P3 - R (mm)	P4 - Total Deformation Reported Frequency (Hz)
2	1 DP 0	10.5	5	5	558.1
3	2	10.5	4.5	4	2149.3
4	4	10.5	5	4.5	1307.2
5	5 DP 3	10.5	5.5	5	559.25
6	6 DP 4	11	4.5	4.5	1268.7
7	7	11	5	5	540.77
8	8	11	5.5	4	2073.7
9	9	11.5	4.5	5	526.33
10	10	11.5	5	4	2022.6
11	11	11.5	5.5	4.5	1248.5

Fig. 4 *Taguchi orthogonal array L9 in ANSYS*

4 ANOVA ANALISYS

Analysis of variance (ANOVA) represents the core of the statistical evaluation of the experiment: it decomposes the total variability of the output into contributions originating from individual parameters and the residual (random) error. In Minitab, the General Linear Model (GLM) was applied with the three selected numerical parameters h, b, and R and the response variable Frequency (the first natural frequency). The model was fitted as a linear model, and Minitab computed the standard ANOVA metrics: sum of squares (SS), mean squares (MS), F-ratio, and p-values, along with coefficient estimates and model quality indicators (R^2).

The resulting regression equation is:

$$f = 9134 - 72.8 \times h - 21 \times b - 1539.8 \times R \qquad (1)$$

The signs of the coefficients indicate the direction of influence: increasing any of the parameters reduces the frequency. The effect of R is several times stronger than that of h, while the influence of b is very small.

The table with results of ANOVA analisys confirms this trend (Table 2). Based on the p-values, R is extremely significant ($p < 0.001$), h is also significant ($p = 0.032$), while b does not show statistical significance within the selected range ($p = 0.437$). More importantly for engineering decision-making, the Contribution (%) quantifies the practical share of variability:

$R \approx 99.63\%$, $h \approx 0.22\%$, $b \approx 0.02\%$, while the residual error is only 0.13%.

Therefore, nearly all frequency variability originates from the notch radius, with a small secondary influence of thickness, whereas the width b does not affect the result in a measurable way within this range. The model quality is excellent: $R^2 = 99.87\%$ (Adj $R^2 = 99.79\%$), and the standard error of the estimate is approximately $S \approx 30$ Hz.

5 MONTE CARLO SIMULATION

Monte Carlo (MC) simulation is a standard statistical technique for quantifying uncertainty when a system is described by a model whose inputs are not fixed values but random variables. Instead of producing a single "point" estimate, MC provides a distribution of possible outcomes, enabling the calculation of mean values, dispersion, percentiles, and probabilities of exceeding specification limits. The principle is simple: the inputs are randomly "drawn" from their probability distributions many times, the model computes the output for each draw, and the resulting outcomes are then used to build descriptive statistics [11].

The method begins by selecting the input distributions—here, the chosen geometric parameters. These distributions must reflect the physical source of uncertainty: a normal distribution when the value results from the accumulation of many small deviations, or a uniform distribution when a strict "min–max" interval is known without assumptions about the center. If geometric or technical constraints exist (e.g., a relationship between parameters), they are incorporated into the MC simulation either as logical conditions or by restricting the sampling space so that constraint violations become negligibly rare [12-13].

The second required component of the simulation is the transfer function. In this study, it is the prediction equation obtained through ANOVA analysis, which replaces expensive numerical simulations and allows thousands of iterations to run efficiently. For each realization of the input

variables, the model computes the output, and after N iterations, the results are statistically processed. The accuracy of MC estimates increases with the number of iterations, and 10,000–50,000 iterations typically yield stable values for most engineering applications [14].
The simulation was conducted in Minitab (Workspace Software): the input distributions were defined, the prediction equation was implemented, and many iterations were executed. An additional output variable, Valid = if (h > 2R, 1, 0), was included to verify that the geometric constraint h > 2R is satisfied in all iterations (Fig. 5).

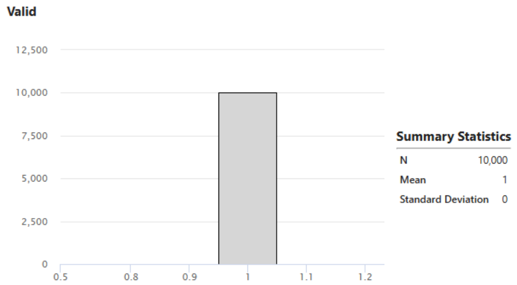

Fig. 5 *Validity Indicator Histogram*

The resulting frequency (Freq) histogram (Fig. 6) is unimodal and approximately symmetric around ~565 Hz, consistent with the numerical results: Mean = 565.255 Hz and Median = 564.759 Hz (nearly identical), with StDev = 78.187 Hz. This implies that roughly 68% of outcomes fall within 487 - 643 Hz, while 95% lie in the P5–P95 = 436.95–692.98 Hz range (and 90% in 466.55–665.66 Hz). The extremes (min = 233.127 Hz, max = 852.079 Hz) are rare and do not distort the overall shape of the distribution.
The symmetry of percentiles around the mean, together with the agreement between the empirical P5 - P95 half-range (~77.9 Hz) and the calculated standard deviation (78.19 Hz), indicates that the distribution is effectively normal. Additionally, the MC mean value agrees with the ANOVA/GLM model prediction evaluated at the midpoints of the inputs (~565.6 Hz), confirming the consistency of the model.

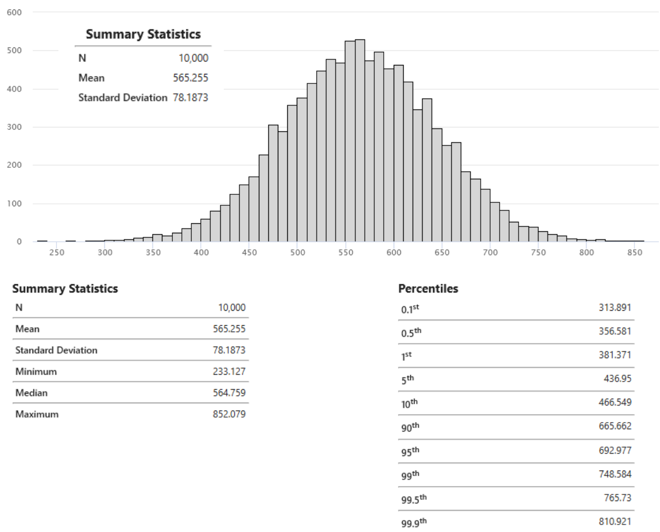

Fig. 6 *Monte Carlo simulation results*

6 CONCLUSION

This study demonstrated a complete uncertainty-analysis workflow for circular-notch flexure hinge: from cost-efficient experimental planning using the Taguchi L9 design, through statistical identification of influential parameters via ANOVA, and finally quantitative prediction of system behavior using Monte Carlo simulation. The DOE → ANOVA → MC chain proved to be highly effective: with only nine FEA evaluations, a clear and linear model of the first natural frequency was obtained, with excellent goodness of fit ($R^2 \approx 99.8\%$).
The results are both consistent and intuitively aligned with engineering expectations. ANOVA revealed that the notch radius R is by far the dominant factor (≈99.6% contribution to variance), the thickness h has a smaller but statistically significant effect, while the width b is essentially negligible within the examined range. The signs of the regression coefficients confirm intuition: increasing the radius and thickness decreases the natural frequency, with the effect of the radius being several times stronger. The Monte Carlo simulation, based on the derived prediction equation and realistic input distributions, produced an output distribution centered around 565 Hz with a standard deviation of about 78 Hz, in full agreement with the deterministic ANOVA/GLM prediction for mid-range input values. This further validates the robustness of the overall methodology, as the ANOVA conclusions are mirrored in the stochastic representation of the system.
These findings lead to clear recommendations for design and manufacturing. In practice, the greatest attention must be directed toward controlling the notch radius, as this parameter almost entirely dictates the natural frequency of the joint. The dimension b may be adjusted more freely—based on criteria such as strength, assembly, available space, or stiffness in other directions—without meaningfully altering the dynamic response. The ANOVA contributions (R ≈ 99.6%, h ≈ 0.2%, b ≈ 0.02%, residual error ≈ 0.13%) indicate that the selected geometric parameters fully explain the variability within the studied domain; no "hidden" geometric factor appears to contribute additional variance.
Further practical implications also emerge from this study. The integration of Taguchi L9 design and ANOVA analysis enabled the construction of a reliable regression model using only nine simulations instead of twenty-seven, significantly reducing computational cost without sacrificing accuracy. For future investigations of this class of compliant joints, three-dimensional simulations are not strictly required—2D (Planar) ANSYS models offer sufficient accuracy and can accelerate computation by a factor of 10 to 50, enabling the exploration of millions of parameter combinations. A natural continuation of this work involves experimental validation of the obtained coefficients and the inclusion of material variability (E, ρ) in Monte Carlo analysis to build an even more robust predictive model.
Overall, the presented methodology provides a reliable, economically efficient, and engineering-transparent framework for designing compliant mechanisms, offering clear guidance for geometric optimization and quality control in production.

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REFERENCES

1. Howell, L. L., 2001, *Compliant Mechanisms*, Wiley, New York.
2. Platl, V., Zentner, L., 2024, An analytical approach for calculating the first natural frequency of flexure hinges with variable cross-sections for compliant mechanisms, *Advances in Mechanism and Machine Science*, IFToMM WC 2023, Ed. Okada, M., Springer, Cham, Vol. 149, pp. 491-501.
3. Šalinić, S., Nikolić, A., 2017, Determination of natural frequencies of a planar serial flexure-hinge mechanism using a new pseudo-rigid-body model (PRBM) method, *Proceedings of the 6th International Congress of Serbian Society of Mechanics*, Mountain Tara, Serbia, S3a, June 19–21.
4. Vu, N. C., Huynh, N. T., Huang, S. C., 2019, Optimization of the first frequency modal shape of a tensural displacement amplifier employing flexure hinge by using Taguchi method, *Journal of Physics: Conference Series*, 1303(1), 012016.
5. Wang, C. N., Truong, K. P., Huynh, N. T., Hoang, L. Q. N., 2019, Multi-parameter optimization of compliant mechanisms, *Journal of Physics: Conference Series*, 1303, The Second International Conference on Mechanical, Electric and Industrial Engineering, Hangzhou, China, 012063.
6. Dao, T. P., Huang, S. C., 2017, Optimization of a two degrees of freedom compliant mechanism using Taguchi method-based grey relational analysis, *Microsystem Technologies*, Vol. 23, pp. 4815–4830.
7. Montgomery, D. C., 2017, *Design and Analysis of Experiments*, 9th ed., Wiley, New York.
8. Roy, R. K., 2010, *A Primer on the Taguchi Method*, Society of Manufacturing Engineers, Dearborn, MI.
9. Baligidada, S. M., Chandrasekhar, U., Elangovan, K., Shankar, S., 2017, Taguchi's approach: design optimization of process parameters in selective inhibition sintering, *Procedia Manufacturing*, Elsevier Ltd., Vol. 5(2), pp. 4778-4786.
10. Reh, S., Beley, J.-D., Mukherjee, S., Khor, E. H., 2006, Probabilistic finite element analysis using ANSYS, *Structural Safety*, Vol. 28(1–2), pp. 17–43.
11. Raychaudhuri, S., 2008, Introduction to Monte Carlo simulation, *Proceedings of the Winter Simulation Conference*, pp. 91–100.
12. Papadopoulos, C. E., Yeung, H., 2001, Uncertainty estimation and Monte Carlo simulation method, *Measurement*, Vol. 12(4), pp. 291–298.
13. Druesne, F., Boubaker, M. B., Lardeur, P., 2014, Fast methods based on modal stability procedure to evaluate natural-frequency variability for industrial shell-type structures, *Finite Elements in Analysis and Design*, Vol. 89, pp. 93–106.
14. Amelin, M., 2015, *Monte Carlo Simulation in Engineering*, KTH Royal Institute of Technology, Stockholm.

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